Massachusetts Professional Content Standards for Adult Mathematics and Numeracy:
What Adult Basic Education Instructors Need to Know and Be Able to Do
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Acknowledgements

Massachusetts Professional Content Standards for Adult Mathematics and Numeracy: What Adult Basic Education Instructors Need to Know and Be Able to Do (herein referred to as *MA Professional Content Standards for Adult Mathematics and Numeracy*) incorporates elements from the following documents:

1. *Massachusetts Adult Basic Education Curriculum Frameworks for Mathematics and Numeracy,* Adult and Community Learning Services, Massachusetts Department of Education, (2005)
2. *Understanding and Accessing the MAPT Score Reports,* Center for Educational Assessment Report No 736., University of Massachusetts Amherst, School of Education and Adult and Community Learning Services Office, Massachusetts Department of Elementary an Secondary Education, (2010)
3. *Guidelines for Mathematical Preparation of Elementary Teachers,* Massachusetts Department of Education, (2007)
4. *College and Career Readiness Standards for Mathematics,(2009)*
    i. *http://opi.mt.gov/pdf/CCSSO/DraftStandards_Math1.pdf*

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Massachusetts Professional Content Standards for Adult Mathematics and Numeracy: Introduction

Context and Purpose

Consistent with the most recent goals set for the state adult basic education system, the purpose of the *Massachusetts Professional Content Standards for Adult Mathematics and Numeracy* is to provide a foundation for building the capacity of ABE and ESOL instructors and strengthening the quality of instruction in adult classrooms. The term 'numeracy' reflects the ability to use numbers in much the same way as literacy reflects the ability to use words. While differing in phrasing and emphasis, the definitions (of numeracy) recognize that mathematics and numeracy are related but are not synonymous. (Ginsburg, Manly, & Schmitt, 2006) Being "numerate" is 'to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life'[1]

In 2009, the Department of Elementary and Secondary Education (ESE) released *Facing the Future: Massachusetts Strategic Framework for Adult Basic Education,*[2] which identified three strategic goals to guide its work:

1. Ensure that adults needing basic education have access to services;
2. Increase system effectiveness and quality; and,
3. Prepare students for success in their next steps: in college and further training, at work, and in the community.

A key objective identified under Goal 2, *increase system effectiveness and quality*, was to strengthen Massachusetts' standards-based ABE system. Part of the plan for meeting the objective was to align standards for instruction and assessment, found in the *Subject Matter Knowledge Requirements*[3] and *Professional Standards for Adult Basic Education Teachers,* [4] with content standards in the *Massachusetts Adult Basic Education Curriculum Frameworks for Mathematics and Numeracy.* [5] In fiscal year 2010, ESE commissioned this document toward that end.

[1] Australian Association of Mathematics Teachers. Numeracy = Everyone's Business. The report of the Numeracy Education Strategy Development Conference. Commonwealth of Australia, Canberra. 1997
[2] http://www.doe.mass.edu/acls/sp/FacingTheFuture.pdf
[3] http://www.doe.mass.edu/lawsregs/603cmr47.html?section=07&flag=
[4] http://www.doe.mass.edu/lawsregs/603cmr47.html?section=08&flag=
[5] http://www.doe.mass.edu/acls/frameworks/

Needs of Students

To fully appreciate the need for professional standards, it is useful to review the problem that they address. Adult learners' challenge with respect to mathematics may be summarized this way:

> *As quantitative and technical aspects of life become more important, adults need higher levels of numeracy to function effectively in their roles as workers, parents, and citizens. The increased need for numeracy skills is amplified by results from recent large-scale surveys of the adult population that indicate that a strikingly large proportion have inadequate skills for the numeracy demands of the twenty-first century.*[6]

Despite these increasing demands, the most recent statewide assessment of adult quantitative literacy revealed that a full 46% of Massachusetts adults were in "basic" or "below basic" levels in quantitative literacy. That statistic implies that nearly half of Massachusetts adults would be unable to calculate the total cost of ordering specific office supplies from a catalogue, for example. [7] Although the study was conducted in 2003, there is no reason to believe that that proportion has changed since that time.

Environmental surveys of the ABE/ESOL programs within Massachusetts conducted from 2006 through 2008 indicate that many beginning learners and most ESOL learners are still not being instructed in math.[8] Local providers tend not to require numeracy skills for ABE/ESOL instructors. Such a lack of emphasis on mathematics and mathematics instruction skills surfaces in student performance outcomes on the GED and ACCUPLACER tests and in college math readiness.[9] While the rate at which GED recipients enroll in postsecondary education appears to be rising, a very high percentage of those who enroll– 88% – are still not graduating.[10] [11] The GED Testing Service reports that the Mathematics Test remains the most difficult for GED candidates.[12] It continues to present a major challenge when the student enters college.

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[6] Ginsberg et al, *The Components of Numeracy,* National Center for the Study of Adult Learning and Literacy (NCSALL) Occasional Paper, 2006

[7] Baer and Hsu, *Highlights from the 2003 Massachusetts State Assessment of Adult Literacy,* American Institutes for Research, http://www.eric.ed.gov/PDFS/ED492900.pdf

[8] Donovan, P "A Peek Inside Massachusetts ABE Classes: How and What Do They Teach?" *Field Notes,* Fall/Winter 2006: 7 http://sabes.org/resources/publications/fieldnotes/vol16/fn161.pdf

[9] Donovan, p. 10

[10] Kroll, *A Promise of Empowerment: Results of the GED 1992 Follow-up Survey*, American Council on Education, Center for Adult Learning and Education Credentials,1995

[11] Patterson et al, *Crossing the Bridge: GED Credentials and Postsecondary Educational Outcomes,* GED Testing Service of the American Council on Education, 2010

[12] GED Testing Service of the American Council on Education, *2008 GED Testing Program Statistical Report*, 2009

Anecdotal reports from Massachusetts' ABE-to-Community College Transition programs; for example, tell us that inadequate math preparation is the greatest barrier to transition to and persistence in post-secondary education. While factors other than quality of instruction contribute to the poor math performances of adult students, it is clear that ABE/ESOL students need stronger instruction in mathematics/numeracy.

Adults' mathematical skills deficits may affect not only their college experience but also their success in the workplace. In 1997, Dr. Arnold Packer, former Executive Director of the U.S. Labor Secretary's Commission on Achieving Necessary Skills (SCANS) noted that high performance employers hire men and women who:

- are able to set up problems,
- know a variety of techniques that apply to problems,
- understand the mathematical features of problems,
- work with others on problems,
- see how to apply mathematical ideas to problems,
- are prepared for open, unstructured problems, and
- believe in the use and value of mathematics in problem solving.[13]

In order to develop these skills, adult students need prepared teachers. These *Standards* are intended to describe a prepared teacher of mathematics and numeracy for adult students: i.e., to increase awareness of the need for professional and proficient instructors of mathematics/numeracy within the adult education community, and to guide teachers, program directors, professional development providers, funders, and other stakeholders in taking the necessary measures to respond effectively.

Needs of Educators

Adult educators today face an unprecedented challenge: they must prepare all adult learners, both native and non-native speakers, to participate in a world with increasing numerical demands. The decades-long effort to establish and improve literacy skills must now be greatly expanded to improve mathematics and numeracy skills. ABE/ESOL programs and

[13] Packer, Arnold, "Mathematical Competencies that Employers Expect," in Madison and Stein, editors, *Why Numbers Count: Quantitative Literacy for Tomorrow's America*, **1997**

teachers hoping to prepare adults for "next steps" involving college and workplace training must pay more attention to the teaching and learning of mathematics and numeracy. In fact:

> *All stakeholders — including policymakers, program directors, educators, professional developers and curriculum designers— need a full understanding of numeracy to know how to provide adults with effective numeracy instruction.*[14]

Just as the *Massachusetts Adult Basic Education Curriculum Framework for Mathematics and Numeracy*[15] provides teachers with standards, benchmarks and examples that describe what adult learners need to know and be able to do in four areas – Number Sense; Patterns, Functions, and Algebra; Statistics and Probability; Geometry and Measurement – the *Professional Standards for Mathematics and Numeracy* outlines what teachers of adult learners need to know, be able to do, and be able to teach in these areas.

Beyond outlining what content and skills adult educators need to master, these professional content standards also address areas of content-specific pedagogy necessary for effective mathematical instruction of adult learners. Both content and pedagogy are addressed because the two are inseparable:

> *Teachers' capacity to pose questions, select tasks, evaluate their pupils' understanding, and make curricular choices all depend on how they themselves understand the subject matter.*[16]

A "full understanding of numeracy" involves not only algorithmic and computational skills, but also ways of thinking about and approaching problem solving.

ABE teachers are expected to attain, over time and through multiple professional development experiences, proficiency with and deep understanding of the arithmetic, algebra, geometry, and probability that their own students will be expected to master in ABE GLE Levels 1-12.[17]

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[14] Ginsberg et al, *The Components of Numeracy,* National Center for the Study of Adult Learning and Literacy (NCSALL) Occasional Paper, 2006

[15] http://www.doe.mass.edu/acls/frameworks/mathnum.pdf

[16] McDiarmid et al, *Why Staying One Chapter Ahead Doesn't Really Work: Subject-Specific Pedagogy*, in M. Reynolds, *Knowledge Base for Beginning Teachers*, Pergamon Press,1989

[17] This paragraph and the bullets that follow it are adapted from the *Massachusetts Guidelines for the Mathematical Preparation of Elementary Teachers*, Massachusetts Department of Elementary and Secondary Education, 2007. http://www.doe.mass.edu/mtel/mathguidance.pdf#search=%22mathematical%22

They can reach this level of knowledge if they:

- come to view arithmetic and algebra as a unified, coherent and consistent subject that all makes sense,
- appreciate the importance of developing clear, explicit, level-appropriate definitions and using logical reasoning to arrive at unambiguous conclusions,
- experience and do real mathematics, by struggling with problems that have multiple steps, logical challenges, and non-obvious solutions,
- acquire habits of mathematical thinking: reasoning, conjecturing, visualizing, analyzing, estimating, exploring, justifying, and constantly probing with "Why?"
- traverse many levels of abstraction: from hash marks on paper to number forms to scientific notation; from numbers to variables to functions, and
- gain the competence and confidence to analyze students' mathematical thinking and engage them in productive mathematical discourse.
- encourage math practice within multiple disciplines such as reading, writing instruction and other content areas.

These *Professional Standards for Mathematics and Numeracy* detail the content knowledge that teachers at various levels of instruction need to possess; and they outline the mathematics teaching skills teachers need to instruct effectively.

By outlining the content knowledge and content-specific pedagogy that adult educators need in order to help students master the content, these standards are intended to be used to:

- guide teachers, program directors, and other stakeholders in knowing what knowledge and skills they should assess and develop, what to seek in hiring new staff and what to develop among veteran staff,
- guide the professional development of ABE/ESOL instructors in ways that align with the *Massachusetts Adult Basic Education Curriculum Frameworks for Mathematics and Numeracy*,
- increase the professional effectiveness of Massachusetts ABE instructors,
- result in students receiving more effective math/numeracy instruction, and

- increase the deliberate preparation of students for “next steps” in education and careers, steps that often rely on sufficient mathematics/numeracy understanding.

The *Subject Matter Knowledge Requirements and Professional Standards for Adult Basic Education Teachers* originated with the ABE licensure process. While they will continue to be used within that context, the use of these requirements and standards will likely be extended to new contexts. The *ABE Professional Standards for Mathematics and Numeracy* are intended to be a multi-purpose tool. The standards may be used by both new and experienced teachers to assess their own areas of strength and weakness and identify professional development goals. Directors and other stakeholders may use them to evaluate teacher mastery and effectiveness and to inform strategies to improve classroom teaching and educational effectiveness.

Professional Standards for Mathematics and Numeracy: Mathematical Teaching Skills

ABE teachers should be able to exemplify the Core Concepts, Guiding Principles and Habits of Mind first outlined in the *Massachusetts Adult Basic Education Curriculum Frameworks for Mathematics and Numeracy* within their math classrooms.

The **Core Concepts** are framed in terms of the curriculum; what teachers should be able to help their students to do.

The **Guiding Principles** are framed in terms of the teacher and what teachers should understand and be able to do.

The **Habits of Mind** are framed in terms of the student and what teachers should exhibit and foster in their classrooms.

A strong math program will:

- appropriately pace and sequence curriculum touching on all math content strands,
- be based on lesson plans which recognize scaffolding opportunities and build on prior knowledge,
- address student goals by incorporating real world mathematics,
- move from the concrete to the abstract,
- apply multiple representations to problem solving,
- utilize varied assessments that inform instruction and are discussed with students,
- provide opportunities for applying creative and critical thinking skills and mathematical discourse,
- utilize appropriate technologies,
- encourage math practice throughout the content areas, and
- validate students using a variety of solution methods.

It is not enough for teachers to know how to perform calculations or solve math problems; they must be able to bring math alive for adult students, to help them see its importance and relevance, its elegance and usefulness, and its fun. The Concepts, Principles, and Habits of Mind set the standards for this level of quality math instruction.

Professional Standards for Mathematics and Numeracy: Core Concepts

These standards support the integrity of the adult learner by encouraging teachers to help their students to reach their own solutions to problems and to reflect and clarify their own thinking about mathematical outcomes. They further require teachers to go beyond the workbook page to help students apply mathematical thinking to other disciplines, as well as in the real world and work-related settings.

Problem solving is an important key to independence for adults.
A teacher should be able to help their students to:

- organize information,
- analyze a problem and make a plan to solve it,
- reach their own solutions,
- generalize problem solving strategies to a wide range of significant and relevant problems, and
- use appropriate problem solving tools including real objects, calculators, computers, and measurement tools.

Mathematical reasoning provides adults with access to information and the ability to orient themselves to the world.
A teacher should be able to help their students to:

- validate their own thinking and intuition,
- pose their own mathematical questions,
- evaluate their own arguments,
- feel confident as math problem solvers,
- determine the degree of precision required by a situation,
- define and select data to be used in solving a problem, and
- apply knowledge of mathematical concepts and procedures to figure out how to answer a question, solve a problem, make a prediction, or carry out a task that has a mathematical dimension.

The ability to **Communicate Mathematically** means having an expanded voice and being heard in a wider audience. A teacher should be able to help their students to:

- interact with others,
- define everyday, work-related or test-related mathematical situations using concrete, pictorial, graphical or algebraic methods,
- express their own thinking about mathematical outcomes, and
- make convincing arguments and decisions based on discussion and reflection.

Connecting everyday life with mathematics helps adults access essential information and make informed decisions, *develop a deeper understanding of mathematics as a whole, and *understand the integral nature of mathematics. A teacher should be able to help their students to:

- view mathematics as an integrated whole that is connected to past learning, the real world, adult life skills, and work-related settings,
- apply mathematical thinking and modeling to solve problems that arise in other disciplines, as well as in the real world and work-related settings,
- *realize connections between mathematical topics such as geometry and algebra or number sense and measurement, and
- *recognize mathematical applications to other areas of study such as sociology, biology, physics, and poetry.

The thinking skills of **Accuracy, Efficiency and Flexibility** are essential tools for success in a rapidly changing world. A teacher should be able to help their students to:

- develop a sense of the appropriate estimate for a solution,
- be able to keep track of how a solution is reached,
- develop the practice of double-checking results,
- use robust strategies that work efficiently for solving different kinds of problems, and
- accept more than one approach to solving a class of problems.

***Representation** of mathematics concepts and problems helps adults become better problem solvers by enlarging their set of mathematical tools, so they can visualize problems in different ways and recognize relationships among mathematics topics.
A teacher should be able to help their students:

- learn to use a variety of mathematical representations, such as number lines, graphs, equations, and pictures,
- apply a variety of mathematical representations to their problem solving efforts, and
- connect mathematical representations in ways that show, for instance, where the coefficient in an equation is visible in a graph.

*Adapted from the National Council of Teachers of Mathematics Principles and Standards.

Professional Standards for Mathematics and Numeracy: Guiding Principles

These Principles call on teachers to exercise instructional strategies that develop specific mathematical practices such as providing access to all math content strands at all levels, as well as more pedagogical practices such as creating problems that accommodate multiple intelligences.

The Guiding Principles summarize a broad vision of adult numeracy that guides all instructional efforts. They address the specific and unique characteristics of both the subject of math and the adult mathematics learner.

Curriculum: A real life context for mathematical concepts and skills across mathematical content strands is the driving force behind curriculum development. Within that setting, mathematics instruction transcends textbook-driven computation practice to include experiences in understanding and communicating ideas mathematically, clarifying one's thinking, making convincing arguments, and reaching decisions individually and as part of a group.
A teacher should be able to:

- describe / implement a contextualized curriculum that
    - incorporates student goals,
    - incorporates real-world mathematics,
    - builds creative and critical thinking skills, and
    - engages students in mathematical discourse.
- *develop / implement an appropriately paced and sequenced curriculum.

Assessment: Mathematical assessment occurs in a framework of purposes for learning relevant to the successful performance of a variety of everyday adult mathematical tasks and the pursuit of further education. Students are active partners in identifying these purposes, in setting personal learning goals, and in defining measures of success.
A teacher should be able to:

- design and use both formal formative and summative assessment tools,
- incorporate informal formative assessment into classroom teaching,
- communicate with students about the purposes, measures, and results of assessments, and
- use assessment results to inform student placement, instruction, and advancement to the next level.

*ANN and NCTM Standards

Equity: Adult numeracy students at every level of instruction and of every gender, sexual orientation, race, religion, and ethnicity require access to all mathematics domains (number sense, patterns, relations and functions, geometry and measurement, probability and statistics).
A teacher should be able to:

- describe ways to promote equity in the classroom,
- use strategies for providing access to all strands at all levels of proficiency,
- recognize and validate strategies and algorithms for solving problems that are based on the diversity of students' cultural backgrounds, and
- provide lessons that incorporate multiple learning styles.

Life Skills: Adult literacy and numeracy education strives to create instruction that helps students become less fearful and more confident in taking risks, voicing their opinions, making decisions, and actively participating in today's world.
A teacher should understand and be able to:

- use instructional approaches that develop students' ability to construct meaning and understanding for themselves,
- know how mathematics is used in a variety of everyday out-of-school tasks, and apply that to lesson planning and instruction, and
- recognize the embedded soft skills in mathematical problem solving e.g., exchanging ideas, listening to other points of view, persevering, creativity, thinking outside the box.

Teaching: Mathematics instruction mirrors real-life activity through the use of both hands-on and printed instructional materials, group as well as individual work, and short-term and long-term tasks.
A teacher should understand and be able to:

- encourage student engagement with mathematics,
- create problems that accommodate multiple intelligences,
- generate and understand multiple solution methods,
- *analyze solutions to determine the source of misconceptions and miscalculation and explain ways to address those errors in understanding and practice,
- model how math is used in real world applications,
- facilitate instruction and discussion in ways that encourage exchange among students,
- facilitate instruction using open-ended questions,

*ANN and NCTM Standards

- facilitate in ways that promote communication of reasoning and justification of solution processes,

- use and understand multiple representations,
- write lesson plans which recognize scaffolding opportunities,
- understand how to scaffold curriculum so as to build on prior knowledge,
- recognize the need of going from the concrete to the abstract,
- use effective questioning based on Bloom's Taxonomy, and
- demonstrate knowledge of the history of mathematics in higher level math classes in order to supplement lessons and connect to social studies, science, and literature.

Technology: Adult numeracy instruction must offer all learners experience with a broad range of technological tools appropriate to a variety of mathematical settings.
A teacher should be able to:

- try new technologies,
- use appropriate technology for problem-solving and instruction e.g., scientific calculators, rulers, protractors, computer programs, and graphing calculators for higher level classes,
- incorporate technology into instruction appropriately, and
- *direct students to online practice and information sites related to areas of study.

*100% Math Initiative: Building a Foundation for Student Success in Developmental Mathematics (Massachusetts Community Colleges Executive Office, 2006)

Professional Standards for Mathematics and Numeracy: Habits of Mind

Teachers are expected to model a curious and open attitude toward new ideas or ways of problem solving, to evaluate reasoning and evidence, to demonstrate persistence and ownership of work, as well as reflection about their problem solving approaches and communication effectiveness.

A teacher should exhibit and foster:

Curiosity
Demonstrate a curious and open attitude towards the presentation of new ideas or ways of approaching problems, even when confusion arises.

Respect for Evidence
Demonstrate the ability to evaluate reasoning because it is essential to see evidence that supports predictions and solutions. Reasoning requires the appropriate use of verbal and visual mathematical evidence to support solutions and ideas.

Persistence
Demonstrate the persistence that is necessary to work through challenging problems that stretch our understanding. Solutions in mathematics are not always apparent at first glance and may involve mathematical risk-taking, exploration, and engagement.

Ownership
Demonstrate taking ownership of our work, which encourages us to do our best and find our own meaning. Although someone else might assign a mathematical task to us, we must treat the problem as important to us, as though it was our own, if we are to produce high quality work and learn from experience.

Reflection
Demonstrate ways to consider how and what we learn from mathematical experiences. To become an autonomous learner, it is necessary to think about how our learning happens.

Professional Standards for Mathematics and Numeracy: Content Strands

There are four Content Strands in the *Massachusetts Adult Basic Education Curriculum Frameworks for Mathematics and Numeracy*.

1. Number Sense
2. Patterns, Functions, and Algebra
3. Statistics and Probability
4. Geometry and Measurement

For each of these Strands, teachers are expected to possess specific knowledge and skills. The overarching ideas they are to know are outlined in introductory pages to each of the Strands.

Format of the Content Knowledge Charts

Standards from the *Massachusetts Adult Basic Education Curriculum Frameworks for Mathematics and Numeracy* form the basis for each Content Knowledge Chart which follows. Items with an asterisk **(*)** are considered equally important for ABE math teachers to know and be able to do, but are NOT included in the current Frameworks.

In addition, students' math skills are assessed by the *Massachusetts Adult Proficiency Test (MAPT).* Class Score Reports by Topic and Content Strand based on the *MAPT for Math and Numeracy* are available for individual teachers through Online Web Learning (OWL), Center of Assessment, University of Massachusetts Amherst and Cognos. The table below provides a highlighted row which references where the *MAPT* topics categories can be found in this document.

The National Reporting System (NRS) of the U.S. Department of Education has defined six levels of student knowledge. Knowledge and skill requirements vary according to the level of class taught. That is, an NRS Math level 1 or 2 teacher is not required to know as much mathematics as a GED or Transitions level math teacher. Within this document, three levels of teachers are distinguished:

- NRS Math Levels 1 and 2 correspond to Level 1 Math/Numeracy
- NRS Math Levels 3 and 4 correspond to Level 2 Math/Numeracy
- NRS Math Levels 5 and 6 correspond to Level 3 Math/Numeracy

It is assumed that teachers at each level may not possess all the knowledge necessary to be effective at their appropriate teaching level; however, such gaps inform teachers, program directors, and other stakeholders of areas needing further professional development.

The content strands that follow, beginning on page 21, contain a table organized by Strand, Standard, MAPT Topic,, Math/Numeracy Level and Benchmark. Understandably, teachers of higher level classes are presumed to have the knowledge base of the lower levels. The diagram below describes the layout of the table that depicts this hierarchy.

Strand

Standard (From MA Frameworks)

MAPT subtopic →	**MAPT Topic:**		
	A teacher will be expected to:		
NRS Level →	**Level 1**	**Level2**	**Level 3**
All teachers should know →	Teachers of all levels will have knowledge of this benchmark.		
Distinction at level 1 →	Teachers of all levels will have knowledge of this benchmark. .	Teachers of Levels 2 & 3 will have knowledge of this benchmark.	
Distinction at each level →	Teachers of all levels will have knowledge of this benchmark. .	Teachers of Levels 2 & 3 will have knowledge of this benchmark.	Teachers of Level 3 only will have knowledge of this benchmark.
Not required at Level 1 →		Teachers of Levels 2 & 3 will have knowledge of this benchmark..	Teachers of Levels 2 & 3 will have knowledge of this benchmark..
Required at Level 3 only →			Teachers of Level 3 will have knowledge of this benchmark

Content Strand: Number Sense

Number sense is the foundation of numeracy that begins in childhood and remains essential in all aspects of adult responsibilities. Teachers of adults must be aware that the student may have gaps in knowledge, so time should be given to strengthening the base. This foundation of numeracy is carried throughout all strands of the math content and includes these key areas of focus:

1. *Place value*
   A comprehensive understanding of place value provides the basis for procedural fluency in computation and estimation.
2. *Real Numbers and the Number line*
   Each point on the number line corresponds to a real number, either rational or irrational. The number line can be used to demonstrate operations of numbers, as well as ordering and comparing numbers.
3. *Properties of Numbers*
   The teacher should be familiar with the properties that apply to all numbers and extend into algebra. These properties include the commutative, associative and distributive properties for addition and multiplication. The teacher needs to further be aware that these properties do not apply to the inverse operations of subtraction and division.
   In addition, the teacher must be familiar with the additive identity (0) and inverses and the multiplicative identity (1) and inverses.
4. *Algorithms*
   In the United States, the standard algorithms for addition, subtraction, multiplication and division are taught based on an understanding of place value. The adult education teacher; however, must respect alternate algorithms for subtraction, division, and operations with fractions as taught in Asian, African and Latin American countries.
5. *Equal sign*
   It is essential that the teacher understand that "=" is a statement of numerical equality. Students who have only seen problems presented as 3 + 4 = _____ have come to confuse the equal sign with "get the answer." Establishing the correct meaning of the equal sign in number sense instruction provides a foundation for better understanding in algebra. Students should be given practice filling the blank in statements such as ____ + 4 = 7 or 5 + ____ = 10 - 3.

6. *Estimation and mental strategies*
   In some situations, estimation and approximation may be more appropriate than an exact calculation, e.g., About how many people attended the baseball game? About how much money will I need to take a vacation?. Estimation can also help the student to determine if his answer is reasonable. Teachers must be cognizant of methods used to develop mental strategies to aid in this approximation, e.g., moving the decimal point one place to determine 10%; adding half again to determine 15%.
7. *Proportional Reasoning*
   Understanding the mathematical relationship between two quantities involves proportional reasoning in measurement, rates, ratios and linear functions. Such understanding provides the groundwork for applications in percents, geometry, algebra and statistics.

Professional Standards Content Strand: Number Sense
Teacher Level Responsibilities

Standard N 1 Represent and use numbers in a variety of equivalent forms in contextual situations

***MAPT* Topic: Place Value**			
A teacher will be expected to:			
	Level 1	***Level 2***	***Level 3***
a	Represent and explain whole number and decimal place value using expanded notation.		
b	Use comparison notation such as >, <, and =.		
c	*Recognize the meaning of the equal sign as balancing two equivalent numerical values or expressions.		
d	Read, write, order and compare positive and negative numbers in a practical context.		
***MAPT* Topic: Decimal/Percentage/Fraction**			
A teacher will be expected to:			
	Level 1	***Level 2***	***Level 3***
a	Recognize fractions, decimals and percents as different representations of parts of a whole.		
b	Explain the multiple meanings of fractions, i.e., part of a whole, part of a set.		
c	Represent conversions between fraction, decimal and percent equivalents.		
d	Use and understand the progression of multiple notation methods, e.g., from division box to horizontal division symbol to fraction bar; from multiplication cross to dot and/or parenthesis to number beside variable.		
e	Order and compare whole numbers, fractions, and decimals using various models, such as numeric representations, the number line, table entries or real objects.		
f		Define rational numbers as numbers that can be written as fractions.	Define irrational numbers as non-repeating, non-terminating decimals.
g		Demonstrate an understanding of percentage of increase and decrease.	

***MAPT* Topic: Operations**			
A teacher will be expected to:			
	Level 1	***Level 2***	***Level 3***
a	*Design number problems that develop an understanding of the equivalence relationship – ex. 16 = 9 + 7 sometimes putting the total on the right, sometimes asking what two numbers total the same as another number and a missing number, etc.		
b		Use exponents, exponential and scientific notation.	Use integer exponents ,exponential and scientific notation.
c		Calculate square roots of perfect squares and estimate within range of square root values.	*Calculate the cube root. Calculate the roots of non-perfect squares and non-perfect cubes.
d		*Recognize real world contexts that can be modeled with square roots.	

Standard N 2 Understand meanings of operations and how they relate to one another

***MAPT* Topic: Operations**			
A teacher will be expected to:			
	Level 1	***Level 2***	***Level 3***
a	Demonstrate addition and additive properties (zero, commutative, and associative) and recognize that those properties do not all apply to subtraction.		
b	Demonstrate subtraction as separating and as comparing.		
c	Show that addition and subtraction are inverse operations and that multiplication and division are inverse operations.		
d	Demonstrate multiplication as repeated addition and by an area model.		
e	Demonstrate division as separating into equal groups or the number of equal groups contained within a total.		
f	Demonstrate that zero is the additive identity and that one (in all its forms) is the multiplicative identity.		

g	*Design problems and problem sets that require and/or access the operational properties.	
h	Recognize the operations for which the distributive property applies (multiplication over addition or subtraction).	
i	Recognize notation that represents distributive property in expanded form.	
j	Be able to factor and access different ways to present the concept.	
k	Perform all operations with fractions, decimals, and mixed numbers.	Perform all operations with integers.
l	Demonstrate whole number, fraction and decimal operations with multiple models, including but not restricted to the number line, area, real objects.	Demonstrate integer operations with multiple models, including but not restricted to the number line, area, real objects.

***MAPT* Topic: Decimal/Percentage/Fraction**			
A teacher will be expected to:			
	Level 1	***Level 2***	***Level 3***
a	Explain the differences in results between whole number and fraction/decimal operations.		
b	*Solve percent problems using different representations of parts of the whole.		
c	Express ratios as a comparison of two numbers by division.		
d	Use the three notations for ratios: fractions, colon, and verbal expression.		
e	Demonstrate proportional reasoning through problem solving, using both formal and informal (mental) computations.		
f	Recognize the application of proportional reasoning in varied contexts within mathematics and real life, including percents.		
g	Understand and explain the difference between a ratio and a fraction.		

Standard N 3 Compute fluently and make reasonable estimates

***MAPT* Topic: Place Value**			
A teacher will be expected to:			
	Level 1	***Level 2***	***Level 3***
a	*Explain the role place value plays within operations, both for estimation purposes and for precise calculations.		
b	Access place value understanding and relationships in order to recognize and teach ways to find 10 percent or one percent of any number to subsequently find any percent of any number.		
***MAPT* Topic: Operations**			
A teacher will be expected to:			
	Level 1	***Level 2***	***Level 3***
a	* Explain multiple ways to practice and learn counting, using song, movement, skip counting, and sequential counting.		
b	* Recognize the basic concepts of numbers as quantities and measures, which can only be combined or separated when defined by the same unit.		
c	Demonstrate different strategies for addition such as decomposing numbers, collecting compatible numbers, and applying additive properties.		
d	Demonstrate different strategies for subtraction such as decomposing numbers, borrowing or counting up.		
e	Demonstrate estimation techniques such as rounding and front end operations.		
f	Relate basic arithmetic operations to relevant life contexts.		
g	*Analyze computational and problem solving errors to determine if the source of those errors is procedural or conceptual.		
h	*Use estimation to corroborate results and measure their reasonableness in problems centered on real life contexts.		
i	Recognize the potential and the limitations of calculator use in learning to compute and problem solve.		

Content Strand: Patterns, Functions, and Algebra

Algebra is thought of as a branch of mathematics in which letters or symbols are used to represent unknown values to identify patterns, functions, and exponential relationships. Algebraic thinking, however, is actually introduced in the primary grades when a blank or question mark is used to represent the unknown. Algebra should be introduced as an extension of arithmetic, following all of the same rules for direct and inverse operations. Algebra has been called the "gatekeeper" to higher math[18]. As a result, there is a need for Algebra to be de-mystified and introduced as generalized arithmetic. The key areas of knowledge for teaching Patterns, Functions, and Algebra in the ABE environment include:

1. *Algebraic Thinking*
   According to NCTM Standards (5-8) - Algebra (NCTM, 1989): The student with algebraic thinking will, "Understand the concept of variable, expression, and equation; represent situations and number pattern with tables, graphs, verbal rules, and equations, and explore the interrelationships of these representations; analyze tables and graphs to identify properties and relationships; develop confidence in solving linear equations using concrete, informal, and formal methods; investigate inequalities and nonlinear equations informally; apply algebraic methods to solve a variety of real-world problems and mathematical problems." The teacher must also exhibit these traits.
2. *Variables and Expressions*
   Algebra is based on the use of an alphabetic or symbolic character, commonly termed a variable, to represent an unknown quantity in expressions, equations, and formulas. To evaluate an expression, or a function, the variable, must be assigned, or given, a value. In an equation, the variable is an unknown quantity that can be solved for a specific value(s); its value(s) does not vary beyond the solution. In a formula, e.g., d = rt, the use of alphabetic or symbolic characters can represent specific quantities given in the problem, and the variable which is the unknown must be discerned from the context. The rules of arithmetic and the convention of the order of operations apply to simplifying expressions, as well as, writing and solving equations.
3. *Patterns*
   The teacher needs to present the student with opportunities to identify and describe patterns algebraically. Once the patterns have been described, they can be extended into a prediction or rule.

[18] The Final Report of the National Mathematics Advisory Panel, U.S. Department of Education, 2008 (http://www.ed.gov/MathPanel)

4. *Equations and Inequalities*
   It is essential that the teacher understand that the equal sign, "=", is a statement of numerical equality. An equation is a statement that two expressions are equal and interchangeable. In an equation two expressions are in balance; in an inequality there is imbalance. An inequality compares two expressions indicating which is greater or less than the other. Inverse operations can be used to solve both equations and inequalities.
5. *Functions*
   A function is a relation that maps each element of a set (the domain, x) with one and only one element of another set (the range, y). Examples of algebraic functions, include: y = x [linear function, f(x) = x] and $y = x^2$ [quadratic function, $f(x) = x^2$]. Note that $y^2 = x$ is not a function because the y values corresponding to x will be both positive and negative.
6. *Coordinate axes*
   One representation of algebraic relationships of two variables uses the coordinate axes. Points are plotted to indicate the relationship between a value on the horizontal axis (x) and the vertical axis (y). For a linear function, the slope of the line plotted through these points together with the y-intercept defines the relationship between x and y.

Professional Standards Content Strand: Patterns, Functions, and Algebra
Teacher Level Responsibilities

Standard P-1 Explore, identify, analyze, and extend patterns in mathematical and adult contextual situations

***MAPT* Topic: Patterns**			
A teacher will be expected to:			
	Level 1	***Level 2***	***Level 3***
a	Design or select visual patterns for students to describe in words.	Represent visual and numeric patterns in general rules, equations, tables, and graphs and select or design visual and numeric problems.	
b	Focus attention on what changes and what repeats in a pattern.		
c	Identify prediction as a key use of pattern generalization with whole numbers or objects.	Identify prediction as a key use of pattern generalization with benchmark fractions and decimals, as well as whole numbers or objects.	Identify prediction as a key use of pattern generalization with integers, as well as fractions, decimals, whole numbers and objects.
d	Connect pattern work with real life contexts.		
e			*Represent linear and non-linear functions using function notation, including defining the domain and range of the function.
f			Distinguish between linear and non-linear patterns as represented in graphs, tables, and equations.

Standard P-2 Articulate and represent number and data relationships using words, tables, graphs, rules, and equations

***MAPT* Topic: Algebra**			
A teacher will be expected to:			
	Level 1	***Level 2***	***Level 3***
a	Write an algebraic expression or equation representing a verbal situation or a word problem.		
b	Develop and use formulas from tables with one or two arithmetic steps (interest; Fahrenheit and Celsius temperatures).		
c		Find the rate of change within a table in a linear relationship. Relate that rate of change to an equation or rule and a graph.	
d			*Recognize the meaning of the y-intercept.
e			Understand slope as rise over run.
f			Demonstrate within a graph how the slope of a graph shows itself and relate that demonstration to the formula for finding slope.
g	Locate and plot points on a grid having a horizontal and vertical reference, e.g., street map.	* Plot graphs using a coordinate plane.	Draw graphs by plotting points using graphing calculators or computer software when available or by hand using all quadrants.

Standard P-3 Recognize and use algebraic symbols to model mathematical and contextual situations

***MAPT* Topic: Algebra**			
A teacher will be expected to:			
	Level 1	***Level 2***	***Level 3***
a	Demonstrate the use of a literal term, *n*, to represent a missing value or unknown in addition and subtraction equations.	Distinguish between a literal term, *x,* used as an unknown value, e.g., 5=*x*+1. and used as a variable value, e.g., y=*x*+1.	
b		Articulate connections between arithmetic and algebraic symbol use and equation construction and solutions.	
c		Recognize and use the universally accepted order of operations in numerical and algebraic expressions.	Recognize and use the basic operational rules of arithmetic as they apply to algebra, i.e., combining like terms in polynomials.
d	Explain the importance of inverse operations involving addition and subtraction.	Explain the importance of all inverse operations.	
e		Recognize and articulate the meaning of the equal sign as balancing two equivalent algebraic expressions and/or values.	
f		Use a number line to demonstrate whole number and integer operations.	
g			Represent and solve linear, quadratic equations, exponential and trigonometric equations.
h			*Solve pairs of simultaneous equations in two variables by algebraic or graphical techniques or by inspection.
i	Formulate statements of inequality using whole numbers and benchmark fractions.	Formulate statements that involve inequalities with integers, decimals, benchmark fractions and whole numbers.	

j		* Solve simple inequalities.	
k		* Graph inequalities on a number line.	
l	Create problems involving inequalities using whole numbers and benchmark fractions.	Create problems involving inequalities with integers, decimals, benchmark fractions and whole numbers.	Create problems that require solutions involving inequalities.

Standard P-4 Analyze change in various contexts

***MAPT* Topic: Algebra**			
A teacher will be expected to:			
	Level 1	***Level 2***	***Level 3***
a	Provide opportunities to observe and record qualitative change over time, such as weather or daylight hours.	Explain and demonstrate how a change in one variable (such as time) relates to change in a second variable (such as interest earned).	
b	Provide opportunities to observe and record quantitative change over time, such as savings per month or height and weight gains of children.	Identify, describe, and explain situations with constant *or* varying rates of change and compare them.	
c		Create linear graphs using mechanical or technological means and analyze graphs to determine the nature of changes in quantities represented in linear relationships using appropriate vocabulary.	
d		Explain and demonstrate rates of change as embedded in tabular and graphical representations.	
e		Approximate rate of change from given data and associate that rate of change with slope; calculate slope using the change in *y* over the change in *x* formula.	

Content Strand: Statistics and Probability

Understanding and interpreting statistics is essential for life in today's data-centric world in which much relies on statistical analysis. In order to prepare students for employment, responsible citizenship, and consumer decision-making, a teacher must be able to represent data with the most effective graphical display, interpret them using appropriate statistical measures, and discuss, infer, analyze, and extrapolate both within a set of data and among data sets. A teacher must be knowledgeable in the details of displaying data and in the methods of sample selection.
The key areas of knowledge for teaching statistics and probability in the ABE environment include:

1. *Graphical display*

   Construct and, appropriately, use line plots, stem-and-leaf plots, box-and-whisker plots, bar graphs, and circle graphs to display data. Use line graphs to show the change over time of data.
   Be aware that distortion and bias can be present in graphical displays, e.g., disproportionate size of graphics, omission or inconsistent use of scale, etc.
2. *Data Interpretation*

   Find and interpret measures of central tendency from a data set. Recognize and interpret data that falls into a Normal Distribution, e.g., test scores. Understand the use of standard deviation and how it is calculated.
3. *Informed Decision Making*

   Use statistical data to make decisions as an employee, a family member, and a citizen.
4. *Language of Probability*

   Know that there is a formal language of probability. Understand the difference between theoretical probability (what is expected to happen mathematically in a given situation) and empirical data (what actually happens), and how the empirical tends towards the theoretical in a large sample space. Work effectively with both simple and compound events.
5. *Mathematical Notation*

   Know the notation used in probability and statistics. For example, notation for mean, summation, etc.

Professional Standards Content Strand: Statistics and Probability
Teacher Level Responsibilities

Standard S-1 Collect, organize, and represent data

***MAPT* Topic: Data Usage**			
A teacher will be expected to:			
	Level 1	***Level 2***	***Level 3***
a	Explain the uses and methods of construction for various graph types – histograms (frequency graphs), bar and line graphs, and pie charts.		
b	Pose questions for data collection exercises that engage students and allow for clear categorization.		
c	Understand and explain the difference between quantitative and qualitative data.		
d	Accurately and appropriately represent data using a variety of graph forms.		
e	Demonstrate relationships between data representations, such as tables and line graphs and bar graphs and pie charts.		
f		Determine slope of a graph.	
g		Explain the meaning of slope in a graph.	
h			*Determine the y-intercept of a graph given data in table form or given an equation.
i			Access, demonstrate and explain the use of a spreadsheet.

Standard S-2 Read and interpret data representations

***MAPT* Topic: : Data Usage**			
A teacher will be expected to:			
	Level 1	***Level 2***	***Level 3***
a	Design appropriate questions to analyze data representations.		
b	Accurately read and interpret a variety of data representations using correct terminology and numerical statements.		
c	Establish and explain values represented by horizontal and vertical axes.		
d	*Establish and explain graph increment choices.		
e	Make and request reasonable inferences from data sets and/or data representations.		

Standard S-3 Describe data using numerical descriptions, statistics, and trend terminology

***MAPT* Topic: Data Usage**			
A teacher will be expected to:			
	Level 1	***Level 2***	***Level 3***
a	Know the difference between mean, median, and mode statistics and how to determine each with a given data set.		
b	Know the purpose of stating averages in describing data.		
c		Identify factors that influence mean and median, such as data spread.	
d	Use data terms, such as *minimum, maximum*, *spread, range, and gaps.*		Use data terms, such as *minimum, maximum*, *spread, range, gaps, holes*, and *clusters.*
e			*Recognize the characteristics of a normal distribution and find the standard deviation.

Standard S-4 Make and evaluate arguments and statements by applying knowledge of data analysis, bias factors, graph distortions, and context

***MAPT* Topic: Data Usage**			
A teacher will be expected to:			
	Level 1	***Level 2***	***Level 3***
a	Recognize graph distortions and explain how different graph distortions influence data interpretation.		
b	Identify biased statements generated from data.		
c	Use graph labels, titles, and data quantities when determining validity of data interpretations.		
d	Evaluate statements based on data as plausible or not.		
e	Support arguments based on data and data representations and use numerical statements to indicate the strength or weakness of an argument or claim.		
f	Generate comparative and contrasting statements based on graphical data.		
g	Generate narratives from graphical representations of data and vice versa.		
h	Explain which statistic (mean, median, or mode) is most useful in forwarding an argument based on data.		
i	*Explain the relationship between author biases and conclusions stated based on data.		
j			Recognize that different categorizations of data can result in formation of different inferences and different stories.
k			Recognize, explain and generate ideas demonstrating how data compression influences conclusions based on full data sets in both useful and distorting ways.

l			Explain the influence of sample size and sample composition on the reliability of conclusions reached from data.

Standard S-5 Know and apply basic probability concepts

***MAPT* Topic: Probability**			
A teacher will be expected to:			
Level 1		***Level 2***	***Level 3***
a	Verbally and numerically describe probabilities for independent events.	Verbally and numerically describe probabilities for dependent events.	
b		Determine total combinations for data sets.	
c			*Determine permutations for data sets.

Content Strand: Geometry and Measurement

Geometry provides a way to describe and make sense of the spatial aspects of the physical world. Measurement links Geometry with Algebra and Number Sense. Geometry and measurement lessons offer clear opportunities to make real world connections that tap into the multiple intelligences expressed by ABE students. To effectively develop this instruction, the ABE Instructor must be competent in the following key areas of focus:

1. *Vocabulary*

   A teacher must be familiar with the numerous definitions found in geometry, the use of these definitions in describing physical models and spatial concepts, and the precision necessary in using the vocabulary. The development of concepts includes the colloquial use of vocabulary at the lower developmental levels and ranges through the precise use of geometric terms at the higher levels.
2. *Formulas*

   Formulas bridge geometry to number and algebra. It is important to know and understand that developmental processes lead to the generation of formulas and that formulas must be understood flexibly, so that, for instance, knowing the formula A=LW allows us to work also with the formula W=A/L.
3. *Units of Measure*

   Comfort with units of measure, conversions between units within a formal system of measurement as well as between and/or among systems, and appropriate use of units based on context is required.
4. *Dimensionality*

   In order to construct the knowledge necessary for the student to effectively use geometric formulas, the teacher must understand that perimeter and circumference, area, volume, and surface area measure dimension, and be cognizant of the units that apply to these dimensions, i.e., units of length, units of area (square units), and units of volume (cubic units). Recognize that circles and spheres move thinking beyond linear measurement.
5. *Angles*

   The concept of an angle as a “Sweep of Space”, the opening between the rays forming the angle, is essential knowledge for the Adult Educator. Angles, lines, triangles, and their relationships must be understood, including the relationship among the angles formed when two parallel lines are cut by a transversal, right triangles and the Pythagorean Theorem, as well as trigonometric ratios, and the relationships amongst the sides found in 45-45-90 and 30-60-90 triangles.

6. *Similarity and Congruence*

Identify corresponding sides and corresponding angles in polygons and be conversant in determining whether a congruence or similarity relationship exists.

7. *Proof*

   While formal proofs are associated with classic geometry classes, the ABE teacher should rely more on informal proof, with reasoning and evidence used to support geometric claims, e.g., that a square is a rectangle, or that two lines are parallel

Professional Standards Content Strand: Geometry and Measurement
Teacher Level Responsibilities

Standard G-1. Use and apply geometric properties and relationships to describe the physical world and identify and analyze the characteristics of geometric figures

***MAPT* Topic: Shapes**			
A teacher will be expected to:			
	Level 1	***Level 2***	***Level 3***
a	Use mathematical notation to identify points, lines, line segments, rays, angles and triangles.		
b	*Demonstrate a conceptual understanding of angle measure (a measurement of the "sweep" of space created by the opening of 2 rays with a common vertex) vs. linear measure (distance between 2 points).		
c	Define types of angles e.g., acute, right, obtuse, straight, reflex.		
d	Use a protractor to measure angles.		
e	*Identify angle relationships e.g., adjacent, supplementary, complementary, vertical.		
f	Demonstrate an understanding of parallel and perpendicular lines.		
g	Identify angle relationships when parallel lines are cut by a transversal e.g., alternating interior and exterior angles.		
h	Define types of triangles using angle measure and side relationships e.g., scalene, right, equiangular, isosceles, equilateral.		
i		Know the Pythagorean Theorem.	
j		Demonstrate the Pythagorean Theorem in multiple ways.	
k		Use the Pythagorean Theorem in problem solving and in contexts involving real-world uses.	
l		Use properties of triangles to solve problems.	

m		Use special right triangle relationships e.g., 45-45-90, 30-60-90 to solve problems.	
n			Define trigonometric functions.
o			Use right-triangle trigonometry to model and solve problems.
p			Recognize problems that require trigonometry.
q	Identify plane figures (both regular and non-regular) and all attributes of these figures e.g., length, width, radius, diameter, height.		
r		Identify and classify right geometric solids e.g., polyhedrons, cylinders, prisms, cones, pyramid.	
s			Use mathematical models to solve problems involving plane and solid figures.
t	Compare the measure of attributes of plane figures both visually and mathematically.		
u		Prove congruence and similarity of plane figures.	
v			*Identify tangents lines and use this concept in problem solving.
w	Compare properties of quadrilaterals e.g., parallel sides, equivalent sides and/or angles.		
	***MAPT* Topic: Things to Measure**		
	A teacher will be expected to:		
	Level 1	***Level 2***	***Level 3***
a	Use formulas to find the perimeter/circumference and area of plane figures, both simple and complex.		
b		Use formulas to find total surface area, lateral surface area, and volume of geometric solids.	

c			*Use trig functions of scientific and/or graphing calculator proficiently in problem solving.
d			*Find the measure of chords, central angles, inscribed angles, and arcs, and use these concepts in problem solving.
e	*Recognize the role that proof and/or justification plays in informal geometry.		*Demonstrate the use of formal proof in geometry.

Standard G-2 Use transformations and symmetry to analyze mathematical situations

***MAPT* Topic: Shapes**			
A teacher will be expected to:			
	Level 1	***Level 2***	***Level 3***
a	Identify all lines of symmetry in geometric (plane) figures.		
b	Demonstrate problem solving/real world context using lines of symmetry in geometric (plane) figures.		
c		Use coordinate geometry effectively to describe translations, rotations, *dilations, and reflections of geometric figures.	
d		Draw 2-D figures in different orientations.	

Standard G-3 Specify locations and describe spatial relationships using coordinate geometry and other representational systems

***MAPT* Topic: Shapes**			
A teacher will be expected to:			
	Level 1	***Level 2***	***Level 3***
a	Describe locations on a map using primary direction, secondary direction, and coordinate grid positions.		
b		Make scale drawings of 2-D and 3-D figures.	
c	Demonstrate and use spatial relationships in real life contexts e.g. giving directions, placing/identifying points of interest accurately on a map.		
d		Identify, locate and graph points on a Cartesian coordinate system.	
e		Draw 2-D figures in different orientations on a grid.	
***MAPT* Topic: Things to Measure**			
A teacher will be expected to:			
	Level 1	***Level 2***	***Level 3***
a	Interpret the key and the scale on a map and use these to find distance among locations on the map.		
b		*Find the distance between two points on a Cartesian coordinate system.	
c		Find and use the y-intercept of a line on a Cartesian coordinate system.	
d		Identify and interpret parallel and perpendicular lines.	
e			Identify and interpret parallel and perpendicular lines on a coordinate system.
f		*Find the midpoint of a line segment on a Cartesian coordinate grid.	
g		Use coordinate geometry in real-world contexts e.g., quilt patterns, designing stairways & entry ramps.	

Standard G-4 Understand measurable attributes of objects and the units, systems, and processes of measurement and apply appropriate techniques, tools and formulas to determine measurements

***MAPT* Topic: Things to Measure**			
A teacher will be expected to:			
	Level 1	***Level 2***	***Level 3***
a	Be able to estimate, measure, and compare length, weight, and capacity using the English, metric, and non-formal systems of measurement.		
b		Convert units of measure in the English system.	
c		Convert units of measure in the metric system.	
d	Use appropriate tools of measurement e.g., rulers, weights, graduated cylinders.		
e	Use appropriate units of measure for a given situation.		
f		Know the prefixes of metric units.	
g		Use the prefixes of metric units to describe environment.	
h		Convert units of measure between different systems using both informal methods and formulas.	
i		Solve and estimate solutions to problems involving length, perimeter, area, surface area, volume, angle measurement, capacity, weight, and mass.	
j			Predict the impact of changes in linear dimension on the perimeter, area, and volume of figures.
k	Read, record, use, interpret, and perform calculations with wall clock time in all its incarnations e.g., digital, analog, 12-hr, 24-hr.		
l	Read, measure, and compare Fahrenheit and Celsius temperatures.		

m		Perform conversions between Fahrenheit and Celsius temperatures using formulas.	
n		Demonstrate common, everyday uses of conversions and errors which may occur in the process.	
o		Use the distance = rate x time formula accurately and appropriately.	
p			Approximate, calculate and interpret rates of change from graphical and numerical data.

Appendix

The following document is Appendix C of *Understanding and Accessing the MAPT Score Reports,* written in collaboration between Center for Educational Assessment, University of Massachusetts Amherst and the Adult and Community Learning Services Office, Massachusetts Department of Elementary and Secondary Education. April 2010

Learning More: Connecting Math Instruction to the Massachusetts Adult Basic Education Curriculum Frameworks for Mathematics and Numeracy

Adult educators across the state worked hard to create the *Massachusetts Adult Basic Education Curriculum Frameworks for Mathematics and Numeracy* to guide teachers how to best meet learners' needs in mathematics and numeracy. Since every MAPT for Math item is designed to measure a benchmark specified in the framework, we suggest all adult educators are familiar with these frameworks before interpreting MAPT scores and discussing results with students.

The frameworks can be found at

http://www.doe.mass.edu/acls/frameworks/mathnum.doc

By interpreting students' performance on the MAPT for Math with respect to the *Frameworks*, adult educators can plan instruction around specific topics and benchmarks.

In this appendix we provide a listing of the benchmarks contained in the *Frameworks*, but the benchmarks are abbreviated and so they are not likely to be very informative for determining lesson plans and other instructional interventions. For this reason, the page numbers in the Frameworks where more complete information on the benchmarks can be found are also listed.

Benchmarks by Topic

In this Appendix, we provide a listing of all of the Mathematics and Numeracy benchmarks by their strand/topic classification to aid in interpretation of the Class Score reports. All benchmarks in the Massachusetts Adult Basic Education Curriculum Frameworks are listed here, but some are not measured by the MAPT (i.e., they are not amenable to assessing via a multiple-choice item format).

Strand/Topic *G&M= Geometry and Measurement* *NS= Number Sense* *PFA= Patterns, Functions, Algebra* *S&P= Statistics and Probability*	Benchmark Number	Shortened Benchmark Text	Curriculum Framework Page for Full Benchmark Text and Related Information
G&M: Shapes	2G-1.1	Know properties of 2-D shapes	39
G&M: Shapes	2G-1.2	Recognize shapes in the environment	39
G&M: Shapes	2G-2.1	Find line of symmetry in shapes	39
G&M: Shapes	2G-2.2	Find multiple lines of symmetry	39
G&M: Shapes	2G-3.1	Use compass rose	40
G&M: Shapes	2G-3.2	Use map/directory with coordinate grid	40
G&M: Shapes	2G-4.8	Find perimeter of rectangles	41
G&M: Shapes	2G-4.9	Find area of rectangles	41
G&M: Shapes	3G-1.1	Compare properties of 2-D & 3-D objects	56
G&M: Shapes	3G-1.2	Locate and understand right angles	56
G&M: Shapes	3G-2.1	Estimate line of symmetry in shapes	56
G&M: Shapes	3G-2.2	ID multiple lines of symmetry in shapes	56
G&M: Shapes	3G-2.3	Identify line of symmetry in shapes	56
G&M: Shapes	3G-3.1	Read and use maps/plans	56
G&M: Shapes	3G-3.2	Locate translations of 2-D shapes on grids	56
G&M: Shapes	3G-4.11	ID perimeter as linear & area as sq. units	58
G&M: Shapes	3G-4.3	Read scales w/marked, unmarked labels	57
G&M: Shapes	4G-1.1	Use radius, diameter, circumference of circle	73
G&M: Shapes	4G-1.2	Read measure of angles on protractor	73
G&M: Shapes	4G-1.3	Compare properties of 2-D/3-D objects	73
G&M: Shapes	4G-1.4	Identify congruent/similar shapes	73
G&M: Shapes	4G-1.5	Identify types of angles	73
G&M: Shapes	4G-1.6	Relate angles of parallel lines/transversal	74
G&M: Shapes	4G-1.7	Identify names of triangles by properties	74
G&M: Shapes	4G-1.8	Estimate measure of angle using benchmarks	74
G&M: Shapes	4G-2.1	Estimate line of symmetry in shapes	74

Strand/Topic *G&M= Geometry and Measurement* *NS= Number Sense* *PFA= Patterns, Functions, Algebra* *S&P= Statistics and Probability*	Benchmark Number	Shortened Benchmark Text	Curriculum Framework Page for Full Benchmark Text and Related Information
G&M: Shapes	4G-2.2	Find multiple lines of symmetry in shapes	74
G&M: Shapes	4G-3.1	Read & use scales and distance on map	75
G&M: Shapes	4G-3.2	ID the measure of 3-D shapes drawn to scale	75
G&M: Shapes	4G-3.3	Find translations of 2-D shapes on a grid	75
G&M: Shapes	4G-3.4	Locate points on x-y axes / grid	75
G&M: Shapes	4G-4.8	Find volume of 3 D shapes	77
G&M: Shapes	4G-4.9	Find perimeter/area of combo shapes	77
G&M: Shapes	5G-1.3	Use spatial visualization with geometric figures	90
G&M: Shapes	5G-1.5	Use triangle properties to solve problems	90
G&M: Shapes	5G-1.6	Solve problems with right triangles	91
G&M: Shapes	5G-1.7	Read measure angles on protractor	91
G&M: Shapes	5G-2.1	Use coordinates w/geometric figures	91
G&M: Shapes	5G-4.2	ID effect of linear change on geom. measures	92
G&M: Shapes	5G-4.4	Find area involving inscribed figures	92
G&M: Shapes	6G-1.1	Solve problems with geometric figures	104
G&M: Shapes	6G-1.2	Solve problems with right triangles	104
G&M: Shapes	6G-1.3	Use spatial visualization with geom. figures	104
G&M: Shapes	6G-2.1	Use coordinates of translations & rotations	105
G&M: Shapes	6G-3.1	Give coordinates for translation of shapes	105
G&M: Shapes	6G-4.2	ID effect of linear change on geom. measures	105
G&M: Things to measure	2G-4.1	Total cost and change from whole dollars	40
G&M: Things to measure	2G-4.2	Read digital & analog clocks	40
G&M: Things to measure	2G-4.3	Measurement w/standard, non-standard units	40
G&M: Things to measure	2G-4.4	Measurement Instruments with common units	40
G&M: Things to measure	2G-4.5	Know common unit relationships	41
G&M: Things to measure	2G-4.6	Compare positive Fahrenheit temperatures	41
G&M: Things to measure	2G-4.7	Personal benchmarks of temperature	41
G&M: Things to measure	3G-4.1	+ - x ÷ sums of dollars and cents	57
G&M: Things to measure	3G-4.10	Calculate change in times	58
G&M: Things to measure	3G-4.12	Estimate, compare weights in common units	58
G&M: Things to measure	3G-4.2	Relationships of distance, time, speed	57
G&M: Things to measure	3G-4.4	Read ruler <= 1/8" metric ruler cm, mm	57
G&M: Things to measure	3G-4.5	Compare inches & centimeters	57
G&M: Things to measure	3G-4.6	Convert units of measure in the same system	57

Strand/Topic *G&M= Geometry and Measurement* *NS= Number Sense* *PFA= Patterns, Functions, Algebra* *S&P= Statistics and Probability*	Benchmark Number	Shortened Benchmark Text	Curriculum Framework Page for Full Benchmark Text and Related Information
G&M: Things to measure	3G-4.7	Solve problems w/weight & capacity	57
G&M: Things to measure	3G-4.8	Read positive/negative Fahrenheit temps.	58
G&M: Things to measure	3G-4.9	Use & interpret 24 hour clock	58
G&M: Things to measure	4G-4.1	Convert units of measure in different systems	75
G&M: Things to measure	4G-4.2	Compare Fahrenheit to Celsius	75
G&M: Things to measure	4G-4.3	Relate distance, time, speed	76
G&M: Things to measure	4G-4.4	Read to 1/16" & mm on metric ruler	76
G&M: Things to measure	4G-4.5	Match metric units to images	76
G&M: Things to measure	4G-4.6	Compare grams ounces, liters qts, informally	76
G&M: Things to measure	4G-4.7	Read measurement instruments w/familiar units	77
G&M: Things to measure	5G-1.1	Apply ratio/proportion in familiar situations	90
G&M: Things to measure	5G-1.2	Use metric units to describe environment	90
G&M: Things to measure	5G-1.4	Choose formulas for area & volume	90
G&M: Things to measure	5G-3.1	Find slope, y-intercept, & intersection of lines	91
G&M: Things to measure	5G-4.1	Solve & estimate area weight mass etc…	92
G&M: Things to measure	5G-4.3	Calculate rates of change from data	92
G&M: Things to measure	5G-4.5	Convert between Fahrenheit & Celsius	92
G&M: Things to measure	6G-4.1	Solve & estimate area weight mass etc…	105
NS: Decimal/Percentage/Fraction	2N-1.3	Read & compare ½, ¼ of things	30
NS: Decimal/Percentage/Fraction	2N-1.4	Use 50% = ½	30
NS: Decimal/Percentage/Fraction	2N-3.5	Know ½ of even #s up to 100	34
NS: Decimal/Percentage/Fraction	3N-1.2	Read, write and compare common fractions	42
NS: Decimal/Percentage/Fraction	3N-1.3	Use equivalent forms of common fractions	42
NS: Decimal/Percentage/Fraction	3N-1.4	Read & compare 2 decimal places in context	43
NS: Decimal/Percentage/Fraction	3N-1.5	Fraction, decimal & equivalents for ¼, ½	43
NS: Decimal/Percentage/Fraction	3N-1.8	Relate % to ratio, part of 100	43
NS: Decimal/Percentage/Fraction	3N-2.1	Relate unit fraction to division by whole #	44
NS: Decimal/Percentage/Fraction	3N-3. 9	2 decimal place word probs w/ calculator	46
NS: Decimal/Percentage/Fraction	3N-3.12	Find common % when part/whole is given	47
NS: Decimal/Percentage/Fraction	3N-3.4	Basic calculations w/ $	45
NS: Decimal/Percentage/Fraction	3N-3.6	Find fractional parts of whole # amounts	45
NS: Decimal/Percentage/Fraction	3N-3.7	Use fractions and % to find part of whole #s	45
NS: Decimal/Percentage/Fraction	4N-1.10	Solve 1-step % problems w/ratio, proportion	60
NS: Decimal/Percentage/Fraction	4N-1.11	Use equivalent forms of common fractions	61

Strand/Topic *G&M= Geometry and Measurement* *NS= Number Sense* *PFA= Patterns, Functions, Algebra* *S&P= Statistics and Probability*	Benchmark Number	Shortened Benchmark Text	Curriculum Framework Page for Full Benchmark Text and Related Information
NS: Decimal/Percentage/Fraction	4N-1.3	Compare fractions & mixed #s	59
NS: Decimal/Percentage/Fraction	4N-1.4	Read & compare decimals to 3 places	59
NS: Decimal/Percentage/Fraction	4N-1.5	Use fractions & decimal equivalents	60
NS: Decimal/Percentage/Fraction	4N-1.6	Convert fractions to decimals	60
NS: Decimal/Percentage/Fraction	4N-1.7	Compare simple %	60
NS: Decimal/Percentage/Fraction	4N-1.8	Find simple % of increase, decrease	60
NS: Decimal/Percentage/Fraction	4N-1.9	Find part of whole with fractions, %, decimals	60
NS: Decimal/Percentage/Fraction	4N-2.3	Describe relationships w/ ratios	61
NS: Decimal/Percentage/Fraction	4N-3.1	Round decimals in context	62
NS: Decimal/Percentage/Fraction	4N-3.2	+ - x ÷ decimals to 3 places	62
NS: Decimal/Percentage/Fraction	4N-3.3	Numbers as fractions of each other	62
NS: Decimal/Percentage/Fraction	4N-3.4	Common fractions + - x ÷	62
NS: Decimal/Percentage/Fraction	4N-3.5	Find ratio & direct proportion	63
NS: Decimal/Percentage/Fraction	5N-1.2	Read & compare fractions & mixed #s	78
NS: Decimal/Percentage/Fraction	5N-1.3	Read & compare decimal #s of any size	78
NS: Decimal/Percentage/Fraction	5N-1.4	Compare % , and % of increase/decrease	78
NS: Decimal/Percentage/Fraction	5N-1.5	ID & use equivalent fractions, decimals, %	78
NS: Decimal/Percentage/Fraction	5N-2.1	ID effects of operations w/fractions	79
NS: Decimal/Percentage/Fraction	5N-2.3	Relate unit fraction to division by whole #	79
NS: Decimal/Percentage/Fraction	5N-2.4	Convert = fractions, decimals, %	79
NS: Decimal/Percentage/Fraction	5N-3.1	+, -, x, ÷ any size decimals	80
NS: Decimal/Percentage/Fraction	5N-3.2	Calculate ratio & direct proportion	80
NS: Decimal/Percentage/Fraction	5N-3.3	+, -, x, ÷ using fractions & mixed #s	80
NS: Decimal/Percentage/Fraction	5N-3.5	Compute w/% to solve problems in context	80
NS: Decimal/Percentage/Fraction	6N-1.2	Read, write, compare fractions & mixed #s	93
NS: Decimal/Percentage/Fraction	6N-1.3	Read, write, order & compare decimal #s	93
NS: Decimal/Percentage/Fraction	6N-1.4	Compare %, % of increase/decrease	93
NS: Decimal/Percentage/Fraction	6N-1.5	ID & use equivalent fractions, decimals, %	93
NS: Decimal/Percentage/Fraction	6N-2.1	Recognize assoc. comm. distrib. props.	94
NS: Decimal/Percentage/Fraction	6N-3.1	+, -, x, ÷ decimals to 3 places	94
NS: Decimal/Percentage/Fraction	6N-3.2	Calculate ratio & direct proportion	94
NS: Decimal/Percentage/Fraction	6N-3.3	+, -, x, ÷ using fractions	94
NS: Decimal/Percentage/Fraction	6N-3.5	Compute w/ %	95
NS: Operations	2N-1.5	Count by 2, 5, or 10	31

Strand/Topic *G&M= Geometry and Measurement* *NS= Number Sense* *PFA= Patterns, Functions, Algebra* *S&P= Statistics and Probability*	Benchmark Number	Shortened Benchmark Text	Curriculum Framework Page for Full Benchmark Text and Related Information
NS: Operations	2N-2.1	Different meanings of addition	31
NS: Operations	2N-2.2	Subtraction strategies 2, 3-digit #s	31
NS: Operations	2N-2.3	Relate adding & subtracting to 1,000	31
NS: Operations	2N-2.4	Apply meanings of multiplication #s to 12	31
NS: Operations	2N-2.5	Apply meanings of division	32
NS: Operations	2N-2.6	Relate mult & div of 1, 2-digit #s	32
NS: Operations	2N-3.1	Add 2, 3-digit whole #s	33
NS: Operations	2N-3.3	Subtract 2, 3-digit whole #s	33
NS: Operations	2N-3.4	Multiply 2-digit #s by 1,2,3,4,5,10,11	34
NS: Operations	2N-3.6	Divide 2-digit whole #s by 1-digit whole #s.	34
NS: Operations	2N-3.8	Calculator Check whole # operations	34
NS: Operations	3N-1.7	Read, compute squares & cubes of whole #s	43
NS: Operations	3N-2.2	Relate square to square root	44
NS: Operations	3N-2.3	Relate adding & subtracting, to 1,000,000	44
NS: Operations	3N-2.4	Choose operation for 1-step word problem	44
NS: Operations	3N-2.5	Exponents as repeated multiplication	44
NS: Operations	3N-3.1	Divide by 2, 3 digit whole #s, interpret remainders	44
NS: Operations	3N-3.10	+ & - #s to 1,000,000 & check answers	46
NS: Operations	3N-3.11	Mult, divide by 2, 3-digit #s, check answer	47
NS: Operations	3N-3.2	Calculate w/ 3-digit whole #s	44
NS: Operations	3N-3.3	Multiply, divide whole #s by 10, 100	45
NS: Operations	3N-3.8	Find squares, sq. roots & cubes of whole #s	45
NS: Operations	4N-2.1	Choose operations for multi-step word prob	61
NS: Operations	4N-2.2	Perform multiplication accurately	61
NS: Operations	4N-2.4	Read & compute with exponents	61
NS: Operations	4N-2.5	Find exact or estimated square roots	61
NS: Operations	4N-3.10	+ & - all #s, check answers	63
NS: Operations	4N-3.11	Multiply, divide, check answers	63
NS: Operations	4N-3.6	Use correct order of operations	63
NS: Operations	4N-3.7	Add & subtract integers	63
NS: Operations	4N-3.9	Whole #s, fractions, decimals, % w/ calculator	63
NS: Operations	5N-1.6	Read #s in scientific notation	79
NS: Operations	5N-2.2	Understand effects of operations w/ integers	79
NS: Operations	5N-3.4	+, -, x, ÷ integers in context	80

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NS: Operations	5N-3.6	Whole #s, frac, dec, % w/ calculator	80
NS: Operations	5N-3.7	Determine prime #s <= 100	81
NS: Operations	6N-1.6	Read exponential notation w/ neg. exponents	94
NS: Operations	6N-2.2	Use negative exponent as repeated division	94
NS: Operations	6N-3.4	+, -, x, ÷ using integers	95
NS: Operations	6N-3.6	Whole #s, fractions, decimals, % w/ calculator	95
NS: Place value	2N-1.1	Read, compare 3 digit #	30
NS: Place value	2N-1.2	Identify odd /even # s to 1000	30
NS: Place value	2N-3.2	Estimate in 10s or 100s, total <= 1000	33
NS: Place value	2N-3.7	Estimate by rounding 1st	34
NS: Place value	3N-1.1	Read, compare 6 digit #	42
NS: Place value	3N-1.6	Compare positive & negative #s in context	43
NS: Place value	3N-3.5	Estimate 1st then round to 6 digit total	45
NS: Place value	4N-1.1	Read, compare large #s	59
NS: Place value	4N-1.2	Recognize pos, neg #s in context	59
NS: Place value	4N-3.12	ID place value change when x by 10, 100, 1000	64
NS: Place value	4N-3.13	ID place value change when ÷ by 10, 100, 1000	64
NS: Place value	4N-3.8	Estimate answers to calculations	63
NS: Place value	5N-1.1	Read & compare pos, neg #s, any size	78
NS: Place value	6N-1.1	Order pos, neg #s, any size	93
PFA: Algebra	2P-3.1	Use & interpret +, -, x, ÷, =	35
PFA: Algebra	2P-3.2	Read # sentences with blanks or n for #s	36
PFA: Algebra	2P-3.3	Read inequalities for #s <= 1000	36
PFA: Algebra	2P-3.4	Read pos /neg #s as direction & change	36
PFA: Algebra	2P-3.5	Count #s using # line	36
PFA: Algebra	2P-3.6	Describe + - x ÷ = situations with notation	36
PFA: Algebra	3P-2.1	Identify exp/equations for verbal situations	48
PFA: Algebra	3P-3.1	Use & interpret + - x ÷ =	49
PFA: Algebra	3P-3.2	Read & solve algebraic expressions	49
PFA: Algebra	3P-3.3	Substitute #s for variables and solve	49
PFA: Algebra	3P-3.4	Solve for variables in 1 step equations	49
PFA: Algebra	3P-3.5	Identify #s using # line	49
PFA: Algebra	3P-3.6	Read inequalities for #s <=1,000,000	50
PFA: Algebra	3P-3.7	Read integer change on horiz. & vert. axes	50

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PFA: Algebra	4P-2.1	Choose expression for multi-step word problem	65
PFA: Algebra	4P-3.1	Use & interpret + - x ÷ =	65
PFA: Algebra	4P-3.10	Read statements of Inequality for all integers	67
PFA: Algebra	4P-3.11	Solve multi-step equations	67
PFA: Algebra	4P-3.2	Read & ID number operations for ( ), x, ÷	66
PFA: Algebra	4P-3.3	Use correct order of operations	66
PFA: Algebra	4P-3.4	Substitute for variables in + -expressions	66
PFA: Algebra	4P-3.5	Substitute for variables in x, ÷ expressions	66
PFA: Algebra	4P-3.6	Substitute whole #s in formulas	66
PFA: Algebra	4P-3.7	Understand positive & negative integers	66
PFA: Algebra	4P-3.8	Understand integer addition, subtraction	67
PFA: Algebra	4P-3.9	Use # line to represent values	67
PFA: Algebra	5P-2.4	Identify graphs using different techniques	83
PFA: Algebra	5P-3.1	Solve multi-step equations	83
PFA: Algebra	5P-3.2	Evaluate formulas	83
PFA: Algebra	5P-3.3	Solve linear & quadratic equations	83
PFA: Algebra	6P-3.3	Evaluate formulas & functions	97
PFA: Algebra	6P-3.4	Solve equations & systems of linear equations	97
PFA: Algebra	6P-3.5	Recognize & use direct/indirect variation	97
PFA: Patterns	2P-1.1	Complete repeating # patterns <= 1000	35
PFA: Patterns	2P-1.2	Recognize & create repeating patterns	35
PFA: Patterns	2P-2.1	See patterns in + and X tables from 0-12	35
PFA: Patterns	2P-4.1	ID description of change with words	36
PFA: Patterns	2P-4.2	ID description of change with numbers	36
PFA: Patterns	3P-1.1	Complete whole # sequence w/2 steps	48
PFA: Patterns	3P-1.2	Recognize & create repeating patterns	48
PFA: Patterns	3P-1.3	Identify simple patterns in tables	48
PFA: Patterns	3P-2.2	Identify , use formulas from applied tables	48
PFA: Patterns	3P-4.1	Identify effect of one variable on another	50
PFA: Patterns	3P-4.2	Compare rates of change in situations	50
PFA: Patterns	4P-1.1	Complete # sequences w/2-step progression	65
PFA: Patterns	4P-1.2	Recognize & create repeating patterns	65
PFA: Patterns	4P-1.3	Generalize relationships within tables	65
PFA: Patterns	4P-2.2	Identify & use formulas from tables	65

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PFA: Patterns	4P-4.1	Analyze linear change in graphs	67
PFA: Patterns	5P-1.1	Complete repeating & growing patterns	82
PFA: Patterns	5P-1.2	Identify patterns in graphs & tables	82
PFA: Patterns	5P-2.1	Match equations to graphs	82
PFA: Patterns	5P-2.2	Relate tables, graphs, words, equations	82
PFA: Patterns	5P-2.3	Generalize # patterns	82
PFA: Patterns	5P-2.5	ID linear/non-linear real-world graphs	83
PFA: Patterns	5P-4.1	Interpret rates of change from graph/# data	83
PFA: Patterns	6P-1.1	Extend complex types of patterns	96
PFA: Patterns	6P-1.2	Contrast linear/exponential growth	96
PFA: Patterns	6P-2.1 (6P-3.2)	Convert tables, graphs, words, equations	96
PFA: Patterns	6P-2.2	Show patterns with formulas and graphs	96
PFA: Patterns	6P-2.3	ID graphs using different techniques	96
PFA: Patterns	6P-2.4	ID linear/non-linear real world graphs	97
PFA: Patterns	6P-3.1	Recognize same function in problems situations	97
PFA: Patterns	6P-4.1	Interpret rates of change from graph/# data	97
S&P: Data Usage	2S-1.1	Gather data to answer questions	37
S&P: Data Usage	2S-1.2	Group objects or responses by 1 criterion	37
S&P: Data Usage	2S-1.3	Communicate with list, table, or diagram	37
S&P: Data Usage	2S-1.4	Check addition with subtotals, 2 or 3 digits	37
S&P: Data Usage	2S-2.1	Identify graphs/tables	37
S&P: Data Usage	2S-2.2	Find graphs/tables from external sources	37
S&P: Data Usage	2S-2.3	Find simple information in list/table	37
S&P: Data Usage	2S-2.4	Read values on bar graph <=1000	37
S&P: Data Usage	2S-2.5	Give # comparisons on bar graphs	38
S&P: Data Usage	2S-3.1	Match graphs/tables to statements	38
S&P: Data Usage	2S-3.2	Determine graph connection to a statement	38
S&P: Data Usage	2S-3.3	Support simple data statements	38
S&P: Data Usage	2S-3.4	Identify relative amounts & misstatements	38
S&P: Data usage	3S-1.1	Select data to answer posed questions	51
S&P: Data usage	3S-1.2	Group objects/responses by a criterion	51
S&P: Data usage	3S-1.3	Choose easiest representation of info	51
S&P: Data usage	3S-1.4	Check total through sum of subtotals	51
S&P: Data usage	3S-1.5	Choose correct categorical data on plots	51

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S&P: Data usage	3S-2.1	ID graphs/tables in available resources	52
S&P: Data usage	3S-2.2	Find graphs/tables in external sources	52
S&P: Data usage	3S-2.3	Identify graphs & tables by type	52
S&P: Data usage	3S-2.4	Extract simple info from list/table	52
S&P: Data usage	3S-2.5	Read values on bar/line graph to 6 digits	52
S&P: Data usage	3S-2.6	Identify # comparisons on bar graphs	52
S&P: Data usage	3S-3.1	Identify min, max, spread, shape of data	53
S&P: Data usage	3S-3.2	Use “most of” statements to describe data	53
S&P: Data usage	3S-3.3	Find average (mean) & range for data set	53
S&P: Data usage	3S-3.4	Find median	53
S&P: Data usage	3S-4.1	Match graphs & tables w/statements	53
S&P: Data usage	3S-4.10	Recognize misleading data manipulation	54
S&P: Data usage	3S-4.11	Identify obvious misstatements	54
S&P: Data usage	3S-4.12	Use “double” “half” & 50% statements	54
S&P: Data usage	3S-4.2	Determine relevance of graphs & tables	53
S&P: Data usage	3S-4.3	Identify relative amounts & misstatements	53
S&P: Data usage	3S-4.4	Support simple statements w/data	53
S&P: Data usage	3S-4.5	Estimation (most) to support arguments	54
S&P: Data usage	3S-4.6	Judge statements of double, half, 50%	54
S&P: Data usage	3S-4.7	Know when % figures don’t add to 100%	54
S&P: Data usage	3S-4.8	Use mean & median to describe data	54
S&P: Data usage	3S-4.9	Recognize misleading info w/ bar widths	54
S&P: Data usage	4S-1.1	ID data to answer questions about life	68
S&P: Data usage	4S-1.2	Group objects/responses by 1 or 2 criteria	68
S&P: Data usage	4S-1.3	Choose best graph to represent of info	68
S&P: Data usage	4S-1.4	Use subtotals to check total	68
S&P: Data usage	4S-1.5	Match data to bar graph, fraction pie chart	68
S&P: Data usage	4S-1.6	ID related bar graph & circle graph	68
S&P: Data usage	4S-1.7	ID related table to line graph & vice versa	68
S&P: Data usage	4S-2.1	Identify graphs/tables	69
S&P: Data usage	4S-2.2	Find graphs/tables in external sources	69
S&P: Data usage	4S-2.3	Identify graphs from tables	69
S&P: Data usage	4S-2.4	Extract simple info from list/tables	69
S&P: Data usage	4S-2.5	Read values on a bar/line/circle graph	69

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S&P: Data usage	4S-2.6	Compare relative values on bar/circle graphs	69
S&P: Data usage	4S-3.1	Identify min, max, spread, shape of data	70
S&P: Data usage	4S-3.2	Use "most of" statements to describe data	70
S&P: Data usage	4S-3.3	Find mean	70
S&P: Data usage	4S-3.4	Find median & mode	70
S&P: Data usage	4S-3.5	ID effect of spread on mean & median	70
S&P: Data usage	4S-4.1	Determine if statements connect to graph/table	71
S&P: Data usage	4S-4.2	Identify relative amounts & misstatements	71
S&P: Data usage	4S-4.3	ID statements supporting/rejecting data trends	71
S&P: Data usage	4S-4.4	Evaluate “double” “half” & 50% statements	71
S&P: Data usage	4S-4.5	ID accuracy of statements w/3x, 4x, ¼, 1/10	71
S&P: Data usage	4S-4.6	Verify that numbers and % figures match	71
S&P: Data usage	4S-4.7	Confirm or deny statements with graphs	71
S&P: Data usage	5S-1.1	ID numerical or categorical life questions	84
S&P: Data usage	5S-1.2	Choose data which responds to questions	84
S&P: Data usage	5S-1.3	ID data w/appropriate representation	84
S&P: Data usage	5S-1.4	ID comparative data on a question	84
S&P: Data usage	5S-1.5	Read double bar/line graph	84
S&P: Data usage	5S-2.1	Identify graphs & tables	85
S&P: Data usage	5S-2.2	Know likely locations of graphs & tables	85
S&P: Data usage	5S-2.3	Infer data meaning: gaps clusters comparisons	85
S&P: Data usage	5S-2.4	Match words to bar/line/circle graphs & tables	85
S&P: Data usage	5S-2.5	Compare # values on graphs/tables	85
S&P: Data usage	5S-3.1	ID min/max, range shape & central tendencies	86
S&P: Data usage	5S-3.2	Identify spread effect on mean & median	86
S&P: Data usage	5S-4.1	Choose best graph to support a position	86
S&P: Data usage	5S-4.10	Confirm or deny statements with graphs	88
S&P: Data usage	5S-4.2	Support arguments w/data & representations	86
S&P: Data usage	5S-4.3	Covert graphs to narratives	87
S&P: Data usage	5S-4.4	Confirm or deny arguments via data trends	87
S&P: Data usage	5S-4.5	Find impact of spread on mean, median, mode	87
S&P: Data usage	5S-4.6	ID misleading representations on graphs	88
S&P: Data usage	5S-4.7	Recognize misleading data manipulation	88
S&P: Data usage	5S-4.8	Match graph to data, story	88

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S&P: Data usage	5S-4.9	Impact of data compression on argument	88
S&P: Data usage	6S-1.1	ID numerical or categorical life questions	98
S&P: Data usage	6S-1.2	Choose data which responses to questions	98
S&P: Data usage	6S-1.3	Choose appropriate representation of data	98
S&P: Data usage	6S-1.4	ID comparative data for a question	98
S&P: Data usage	6S-1.5	Read double bar/line graphs	98
S&P: Data usage	6S-16	Know how to use a spreadsheet on a computer	98
S&P: Data usage	6S-2.1	Identify graphs & tables	99
S&P: Data usage	6S-2.2	ID likely locations of graphs & tables	99
S&P: Data usage	6S-2.3	Relate words to all types of graphs & tables	99
S&P: Data usage	6S-2.4	Compare relative values on graphs/tables	99
S&P: Data usage	6S-2.5	Infer meaning from gaps clusters comparisons	99
S&P: Data usage	6S-2.6	Infer consequences related to data outcomes	99
S&P: Data usage	6S-3.1	ID data min max spread shape range	100
S&P: Data usage	6S-3.2	Use "most of" statements to describe data	100
S&P: Data usage	6S-3.3	Find mean	100
S&P: Data usage	6S-3.4	Find median	100
S&P: Data usage	6S-3.5	Identify effect of spread on mean & median	100
S&P: Data usage	6S-4.1	Confirm or deny arguments via data trends	100
S&P: Data usage	6S-4.10	Show different aspects of data	102
S&P: Data usage	6S-4.11	ID impact of data compression on argument	102
S&P: Data usage	6S-4.12	Use graphs as evidence for/against arguments	102
S&P: Data usage	6S-4.13	ID simple sample biases	102
S&P: Data usage	6S-4.2	Evaluate accuracy of % statements	101
S&P: Data usage	6S-4.3	ID mean, median, mode as central tendencies	101
S&P: Data usage	6S-4.4	Determine impact of spread on mean, median	101
S&P: Data usage	6S-4.5	Choose mean or median as appropriate	101
S&P: Data usage	6S-4.6	ID misleading info with bar widths	101
S&P: Data usage	6S-4.7	See scale distortions in research materials	101
S&P: Data usage	6S-4.8	Recognize wedge size distortions	102
S&P: Data usage	6S-4.9	Choose explanation for misleading data	102
S&P: Probability	2S-4.1	Identify events as likely/unlikely	38
S&P: Probability	2S-4.2	Find probability in concrete situations	38
S&P: Probability	3S-5.1	Identify events as likely/unlikely	55

Strand/Topic *G&M= Geometry and Measurement* *NS= Number Sense* *PFA= Patterns, Functions, Algebra* *S&P= Statistics and Probability*	Benchmark Number	Shortened Benchmark Text	Curriculum Framework Page for Full Benchmark Text and Related Information
S&P: Probability	3S-5.2	Basic probability in concrete situations	55
S&P: Probability	3S-5.3	Probability as ratio in multiple forms	55
S&P: Probability	4S-5.1	Identify events as likely/unlikely	72
S&P: Probability	4S-5.2	Basic probability in concrete situations	72
S&P: Probability	4S-5.3	State probability as a ratio fraction	72
S&P: Probability	4S-5.4	Find probability of independent events	72
S&P: Probability	4S-5.5	State probability as a %	72
S&P: Probability	5S-5.1	Probability of independent/dependent events	89
S&P: Probability	5S-5.2	Find # of possible combinations	89
S&P: Probability	6S-5.1	ID events as likely/unlikely	103
S&P: Probability	6S-5.2	Find basic probability in concrete situations	103
S&P: Probability	6S-5.3	Use probability as a ratio fraction	103
S&P: Probability	6S-5.4	Use probability as a %	103
S&P: Probability	6S-5.5	Find probability of indep/dependent events	103