A DEEP DIVE INTO TEACHING DIVISION

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WHY IS DIVISION DIFFICULT?

When it comes to the four basic operations, division is often the one that students find the most difficult. Many adult education students, especially at beginning levels of math, have gaps in their understanding of both when and how to divide. Why does this operation seem to cause more problems than the rest? A lack of fluency in basic multiplication facts can sometimes be a factor, but there is more about division that can cause difficulties.

Addition and multiplication have an important property that makes them easier and more intuitive for many people than subtraction and division. When we add,

$$\mathbf{5 + 6 + 7 + 8} = 26$$

all the numbers being added (5, 6, 7, and 8) are called **addends**. They are all performing the same "role" in the calculation. Because of this, we have a lot of flexibility in how we add them. We can rearrange the order (Commutative Property)

$$8 + 5 + 7 + 6 = 26$$

or regroup them (Associative Property)

$$5 + (6 + 7) + 8 = 26$$

without changing the result.

Multiplication has this same characteristic. The numbers we multiply (in the example below, 2, 3, and 4) are all called **factors**

$$\mathbf{2 \times 3 \times 4} = 24$$

and are interchangeable when it comes to calculations.

$$4 \times 2 \times 3 = 24 \text{ (Commutative Property)}$$

$$2 \times (3 \times 4) = 24 \text{ (Associative Property)}$$

Subtraction and division are different. The numbers used in division, for example, are called the **dividend** and the **divisor**. They have different names because they are performing different "roles" in the calculation.

Dividend		**Divisor**		**Quotient**
20	÷	**5**	**=**	**4**

If we consider division's relationship to multiplication, 5 and 4 can also be seen as factors, with a product (result) of 20.

$$5 \times 4 = 20$$

Because factors are interchangeable, there is flexibility in division

$$20 \div 5 = 4 \text{ and } 20 \div 4 = 5$$

but the flexibility is between the divisor and the quotient, not the dividend (20). The dividend has a unique role in division and can't be exchanged without changing the meaning of the calculation.

For example,

$20 \div 5 = 4$ could model a situation where \$20 is shared equally among 5 people, whereas

$5 \div 20 = .25$ would mean that \$5 is being shared among 20 people, a very different situation with a different result.

Division is also the only one of the basic operations that can produce fractional results and remainders from whole number inputs. Physical/visual representations and considering context are important to help students make sense of the idea of a remainder. For beginning level students, keep remainders as whole numbers with labels ($62 \div 12 = 5$, remainder 2) and leave decimal or fraction representations until later.

TWO MODELS OF DIVISION

Modeling with mathematics (a phrase used in the CCRSAE Standards for Mathematical Practice) means using math to describe the behavior or relationship between values in the real world. An important part of helping students understand division is to help them learn to recognize what types of "behavior" can be modeled with division.

PARTITIVE (EQUAL SHARING)

One situation that can be modeled with division is "equal sharing", which is known as **partitive division** (a word for teachers, not students). In situations where an amount is being shared equally into a certain number of groups, the dividend can be seen as the amount being shared, and the divisor represents the number of groups. For example,

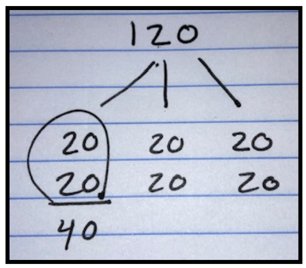
$$\$20 \div 5 = \$4$$

could model a situation in which \$20 was shared equally between 5 people. The quotient (result) tells us how much ends up in each group (in this case, \$4).

Teaching suggestions for partitive division

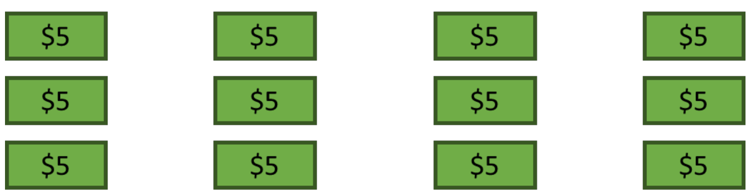
Partitive division can be physically or visually modeled with a "passing out" situation, such as dealing cards, or passing out a pile of money to students equally until it is gone. By distributing the same amount to each group as the dividend is passed out, we can ensure that the distribution is "fair" or equal, and this continues until the dividend is gone. Students should also discover that it is possible to "pass out" value in equal "chunks". For example, I could deal out 3 cards at a time to each player instead of 1.

As students deepen their understanding of creating equal groups, they can start to apply a "passing out" method more representationally. For example, if I wanted to pass out 120 into 3 equal groups, I might give each group 20, then 20 again, until all the value has been distributed.

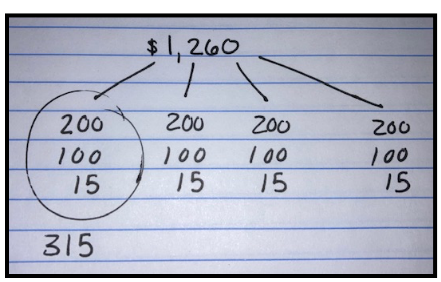


Finances provide a relevant context for exploring and modeling partitive division with adult learners. Splitting a bill, creating a savings plan, figuring out a monthly budget, or determining the monthly payments on a loan all involve partitive (equal sharing) division. Use small dividends for students who still need practice with physical manipulatives, such as money or chips, and larger amounts for folks who can pass out larger chunks by representing the amounts on paper.

More accessible example: A student uses play money to split a $60 bill among four people.



More advanced example: Students use paper and pencil to split a $1,260 loan into four monthly payments.



Because of the passing out nature of partitive division, this is most efficient when there are a small number of groups (in other words, when the divisor is small). It might be a good mental math strategy for a division problem like 270 ÷ 2 because we could break up 270 and pass it out into two groups. However, thinking of 270 ÷ 12 as passing out into 12 groups would be inefficient to do mentally or on paper.

QUOTATIVE DIVISION (HOW MANY ____ IN ____)

I am 62 inches tall. When I want to figure out that height in feet, I am not thinking about passing out my inches into 12 equal groups. Instead, I am wondering how many groups of 12 inches I can make out of 62 inches. This type of situation can be modeled with **quotative division**, which can be thought of as asking, *How many _____ in _____?*

62 ÷ 12 asks how many 12's in 62?

In this case, the dividend is still the total amount, but the divisor (12) represents the size of the group we are interested in. The quotient (5, with a remainder of 2 inches!) tells us how many groups we can make of that size.

Teaching suggestions for quotative division

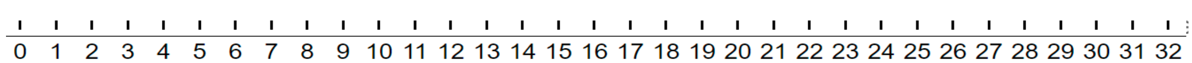
Quotative division lends itself to physical/visual models of scooping and grouping. For example, we could figure out how many 8-ounce cups are in 32 ounces by scooping out 8-ounce cups until there is nothing left. The quotient tells us that we can do this four times (How many 8's in 32? Answer: 4)

$$32 \div 8 = 4$$

Mathematically, quotative division can also be modeled with repeated subtraction of the same amount:

$$32 - 8 - 8 - 8 - 8 = 0$$

Number lines also work well as a visual model for quotative division.



Number lines can also help demonstrate flexible ways of solving quotative division. We can arrive at the quotient of 4 by

1) knowing that 4 x 8 = 32

2) repeatedly subtracting 8's from 32

3) adding 8's from 0 until we get up to 32

Measurements are an important relevant context for adult learners to apply quotative division. When we are converting from smaller units (such as inches) into larger units (such as feet) we are asking how many groups of a certain size we can make (How many groups of 12 inches can I make out of my 62 inches?) This also applies to other common measurements, such as seconds/minutes/hours, months/years, ounces/pounds, etc. Another advantage of using measurements as an application for quotative division is that they easily make sense of remainders (62 inches ÷ 12 = 5 feet, 2 inches)

When calculating mentally or on paper, quotative division is an efficient way to work with large divisors and forms the basis for how most people were taught long division. For example, it is easier to think of 275 ÷ 25 as how many 25's are in 275, rather than envisioning passing out into 25 equal groups. Long division is simply a method that allows us to subtract out groups of 25 and to keep track of how many we have removed.

SOME FINAL THOUGHTS

Division is an operation that builds on the other three (addition, subtraction, and multiplication), and so students should have a conceptual basis and some ability to calculate with the other operations first. However, beginning level students can start to study division concepts even if they have not gained perfect fluency with multiplication facts. Other operations can be used to compensate, and division is an important concept for adults to understand. Work with division can provide more practice working with groups, which will give students more exposure to multiplication at the same time.

As suggested above, it is important for students to learn how division can model familiar contexts, such as paying off a loan or converting measurements. Since adults already have some familiarity with this "behavior", it will help them to understand the more abstract "behavior" of division. Metaphors, such as "equal sharing" and "passing out" (partitive), "How many ___ in a ____", and "scooping and grouping" (quotative) are helpful, as are visual and tactile representations.

Students should explicitly be taught and practice common notation involved with communicating division, including

$$20 \div 2 \qquad \frac{20}{2} \qquad 20 / 2$$

However, the notation used for long division is meant for paper and pencil calculations, not for communication. The setup of long division varies from country to country and students should use whatever format they are comfortable with. If they understand what they are doing, it is not necessary for them to use the U.S. format, as there is no setting outside of a U.S. primary school in which they would be asked to communicate with long division notation.

