



SABES Program Support PD Center

Professional Licensure

**Adult Basic Education
Massachusetts Test for Educator Licensure
(MTEL)**

Math Study Guide

Developed by Allison Reid

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Acknowledgements

Welcome to the *Math Study Guide for the ABE MTEL*. This Guide was developed by Allison Reid to assist Massachusetts adult educators who plan to take the ABE MTEL in pursuit of the ABE Teacher’s License. Allison’s work draws on her experience as an adult educator and License-seeker who herself took and passed the MTEL exams. She also referred to MTEL study materials compiled by Meri Holden at the Attleboro Adult Education Program/Bristol Community College and Karen Miller from the Bristol Community College Literacy Center.

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We hope you will find this Math Study Guide useful as you prepare for your MTEL. We are grateful for Allison’s generous sharing and her enthusiastic support for the ABE Teacher’s License.

We invite you to contact Luanne_teller@worlded.org with questions or comments.

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Planning your Time

A pre-assessment and study planner is included to help you plan your time and track your progress. Consider each topic and your current knowledge and decide whether you are all set, need a quick review or extensive practice. You might also find it helpful (or motivating) to check off each topic as you study it. Textual guidance is supplemented with many online resources.

You likely do not need to do every problem or read every word, at every link, for every topic.

If you are unsure of your skills, try a few and check your answers. If you find you are answering easily, you may be ready to move on to another topic. You might also find that you prefer the format or style of some sites better than others, or that you need more explanation for some topics and more practice than another.

Use the resources that suit you and skip the others... guilt free!

Resources are organized by objectives and topics, and divided into three types:

1. **Interest:** Provide context, different perspectives, application. They're here to keep it interesting so they include history, TED talks, animations etc.
2. **Explanation:** Text or video, these sources try to answer: What does this mean? How do you do this?
3. **Practice:** Your turn to practice applying the skill / answering questions. Interactive or mobile compatible options included.

Blue hyperlinks should take you directly to the online resources.

General Math Study Resources

- GED/HiSET Math: [CBAL Quick Math Practice](#), [HiSET Interactive Practice Tests](#)
- [Khan Academy](#) (Also available as free app for iphone or android) [Using Khan Academy \(video\)](#)
- [Common Symbols in Mathematics](#)

Self-Assessment and Study Planner

Objective 7: Understand number sense and operations

Base 10 system, Basic Operations

- All Set
- Quick Review
- Extended Practice

Order of Operations

- All Set
- Quick Review
- Extended Practice

Exponents & Roots

- All Set
- Quick Review
- Extended Practice

Factors, Prime & Composite Numbers, Inverse

Operations

- All Set
- Quick Review
- Extended Practice

Integers, Negative Numbers, Absolute Value

- All Set
- Quick Review
- Extended Practice

Fractions

- All Set
- Quick Review
- Extended Practice

Decimals

- All Set
- Quick Review
- Extended Practice

Percents

- All Set
- Quick Review
- Extended Practice

Ratios & Proportions

- All Set
- Quick Review
- Extended Practice

Objective 8: Understand basic concepts of algebra

Patterns, Tables & Rules

- All Set
- Quick Review
- Extended Practice

Algebraic symbols & expressions

- All Set
- Quick Review
- Extended Practice

Properties of functions & relations

- All Set
- Quick Review
- Extended Practice

Linear equations

- All Set
- Quick Review
- Extended Practice

Inequalities

- All Set
- Quick Review

- Extended Practice

Systems of equations

- All Set
- Quick Review
- Extended Practice

Quadratic expressions & parabolas

- All Set
- Quick Review
- Extended Practice

Polynomials, FOIL

- All Set
- Quick Review
- Extended Practice

Objective 9 Understand geometry and measurement.

Units and Measurements

- All Set
- Quick Review
- Extended Practice

Area/Perimeter/Volume/Surface Area

- All Set
- Quick Review
- Extended Practice

Angle Relationships, including Opposite/Vertical/Transverse Angles

- All Set
- Quick Review
- Extended Practice

Similar/Congruent

- All Set
- Quick Review
- Extended Practice

Pythagorean

- All Set
- Quick Review
- Extended Practice

Circles

- All Set
- Quick Review
- Extended Practice

Coordinate Geo: points, slope intercept, finding slope

- All Set
- Quick Review
- Extended Practice

Objective 10: Understand data analysis, statistics, and probability

Charts, graphs & tables

- All Set
- Quick Review
- Extended Practice

Central Tendency

- All Set
- Quick Review
- Extended Practice

Statistics: Percentile & Frequency

- All Set
- Quick Review
- Extended Practice

Data analysis: bias factors, graph distortion

- All Set
- Quick Review
- Extended Practice

Probability

- All Set
- Quick Review

Extended Practice

Objective 7: Understand number sense and operations

Base 10 System and Basic Operations

Base 10, refers to our system of numbers in which each place represents a power of 10 (tens, hundreds, thousand etc.) Basic operations include addition, subtraction, multiplication, and division. Sources are listed in parentheses.

Interest

- [From Fish to Infinity](#) (NYTimes)
- [Visualizing One Part per Million](#) (TED Ed)
- [A Brief History of Numerical Systems](#) (TED Ed)
- [How high can you count on your fingers](#) (TED Ed)

Explanation

- [What are Numbers?](#) (skillsyouneed.com)
- [Understanding Place Value](#) (helpingwithmath.com)
- [Estimation, Approximation and Rounding](#) (skillsyouneed.com)
- [Long Division Animation](#) (mathisfun.com)
- [Introduction to Word Problems](#) (GFCGlobal.org)
- [Distance Word Problems](#) (GFCGlobal.org)

Practice

- [Rounding](#) (ipracticemath.com)
- [Estimate sums](#) (mathgames.com)
- [Estimate products](#) (mathgames.com)
- [Distance/Rate/Time Word Problems](#) (BBC)
- [2 Step Word Problems](#) (Khan Academy)

Order of Operations

Order of operations applies when an expression contains more than one operation, e.g., $10 - 2 \times 3$ contains both subtraction and multiplication.

PEMDAS (or Please Excuse My Dear Aunt Sally) is a mnemonic (memory device) to remember the correct order to complete these operations, beginning with **P**arenthesis, then **E**xponents, followed by **M**ultiplication and **D**ivision, ending with **A**ddition and **S**ubtraction.

- **Parentheses:** complete any math contained within parentheses () or other grouping symbols such as brackets { }. If there is more than one operation within the parentheses, complete them in order (e.g., multiplication/division before addition/subtraction.) If there are parentheses within parentheses (called nested parentheses) work from the innermost set out.
- **Exponents:** evaluate any exponents or roots/radicals. (See below for information on calculating exponents)
- **Multiplication and Division:** complete all multiplication and division, moving left to right.
- **Addition and Subtraction:** complete all addition and subtraction, moving left to right.

Explanation

- [Order of Operations](#) (GCFGlobal.com)
- [Introduction to Order of Operations](#) (Khan Academy)

Practice

- [Exploring Order of Operations](#) (Interactive/Requires Flash)
- [Order of Operations: Add, Subtract, Multiply, Divide](#) (iknowit.com)

Mathematical Properties

- **Associative Property:** For addition and multiplication, terms can be regrouped, without changing the final result. Example: $(2 + 3) + 4 = 5 + 4 = 9$ and $2 + (3 + 4) = 2 + 7 = 9$. This property does not apply to subtraction or division. It is frequently written as $a + b = b + a$ and $a \times b = b \times a$
- **Commutative Property:** For addition and multiplication, terms can be re-ordered without changing the final result. Example: $3 \times 4 \times 2 = 24$ and $2 \times 4 \times 3 = 24$. This property does not apply to subtraction or division. It is frequently written as $(a + b) + c = a + (b + c)$ and $(a \times b) \times c = a \times (b \times c)$
- **Distributive Property:** When a number is multiplied by a sum or a difference, it gives the same result as multiplying by each term in the addition or subtraction separately. Example: $2 \times (4 - 3) = 2 \times 1 = 2$ OR $2 \times 4 - 2 \times 3 = 8 - 6 = 2$. This property is frequently used in algebra and is the basis for multiplying polynomials (see Objective 8 for more) It is frequently written as $a \times (b + c) = a \times b + a \times c$

Explanation

- [Properties of Addition](#) (Khan Academy)
- [Commutative, Associative and Distributive Laws](#) (mathisfun.com)
- [What is Distributive Property?](#) (SplashLearn)

Practice

- [The Associative Property](#) (Study.com)
- [Associative Property of Multiplication](#) (Khan Academy)
- [Commutative Property of Multiplication](#) (Khan Academy)
- [Distributive Property](#) (mathgames.com)

Exponents & Roots

An exponent is composed of two parts, a base and an exponent (also called power). In the example 3^4 , 3 is the base, 4 is the power. The base is multiplied by itself, the number of times indicated by the power. That is $3^4 = 3 \times 3 \times 3 \times 3$.

A number to the second power/with an exponent of 2 is commonly referred to as squared, e.g., 5^2 is “five squared”. A third power is “cubed”.

A square root asks what number squared, or times itself, equals the given number, for example: $\sqrt{25} = 5$ because 5×5 or $5^2 = 25$. Many square roots are decimals and are calculated using a calculator button. It may be helpful to memorize the perfect squares (the squares of the numbers 1-10 or 12) as these can be done mentally.

Interest

- [How Folding Paper Can Get You to the Moon](#) (TEDEd)

Explanation

- [Introduction to Exponents](#) (Khan Academy)
- [Finding Square Numbers](#) (K5Learning.com)
- [Understanding Square Roots](#) (Khan Academy)
- [Scientific Notation](#) (Math Antics / YouTube)
- [Exponent Rules with Variables](#) (Math.com)
- [Variables with Exponents](#) (mathisfun.com)

Practice

- [Exponents](#) (Khan Academy)
- [Square Roots of Perfect Squares](#) (mathgames.com)
- [Multiply & Divide Powers](#) (Khan Academy)

Factors, Prime & Composite Numbers

A factor is a number that can be multiplied with another number to produce a third number. The factors of 6 are 1, 2, 3, 6, because $1 \times 6 = 6$ and $2 \times 3 = 6$. Decimals and fractions are generally not included (e.g., $12 \times .5 = 6$, but you wouldn't necessarily refer to .5 as a factor).

Because multiplication and division are inverse operations, factors are also the numbers by which a number can be evenly divided (6 is divisible by 1, 2, 3 and 6 without a remainder). Prime numbers have only themselves and 1 as a factor, e.g., 7 (1×7) or 13 (1×13). Composite numbers have multiple factors, e.g., 6, above).

Explanation

- [Divisibility Rules](#) (IPracticeMath.com)
- [Reciprocals and Inverse Numbers](#) (GFCGlobal.org)
- [Prime and Composite Numbers](#) (mathisfun.com)
- [Prime Factorization](#) (mathisfun.com)

Practice

- [Factor Pairs](#) (iknowit.com)
- [Prime or Composite](#) (Khan Academy)
- [Reciprocals and Multiplicative Inverse](#) (mathgames.comcom)

Integers and Negative Numbers

Integers are all numbers without a fraction or decimal component. They may be positive or negative: e.g., 3, 57, -11 are integers. 2.7, -0.3, $4\frac{1}{2}$ are not.

Negative values are below zero (think of a debt, or a below freezing temperature). Positives are above (such as a positive account balance, or the elevation of a mountain).

Absolute value is the distance from zero in either direction $|-6| = 6$, because it is 6 units from zero. In practice, it removes the positive/negative sign.

- Multiplying/Dividing: An even number of negative values multiplied (or divided) together yields a positive answer, an odd number yields a negative one: e.g., $-2 \times -4 = 8$ (two negatives), $-3 \times 5 = -15$ (one negative).
- Adding: If the signs match, the sign stays the same; simply add the values. If they do not match, the answer will have the sign of the term with the greater absolute value; take the difference between the values, e.g., $-4 + 6 = 2$. Positive because 6 is greater, 2 because that is the difference between the 4 and 6.
- Subtracting: Convert subtraction to addition, by changing the operation and sign of the second term, e.g., $-2 - 4 = -2 + -4$. Use the procedure above.

Interest

- [The History of Negative Numbers](#) (NRICH)

Explanation

- [Negative Numbers](#) (GCFglobal.com)
- [Intro to Negative Number](#) (Khan Academy)
- [Introduction to Negative Numbers](#) (PurpleMath.com)
- [Absolute Value](#) (Math Antics/ YouTube)

Practice

- [Negative numbers, variables, number line \(Khan Academy\)](#)
- [Integer addition and subtraction rules](#) (mathgames.com)
- [Multiplying negative numbers](#) (Khan Academy)
- [Finding absolute values](#) (Khan Academy)

Fractions

Fractions represent parts of a whole. A fraction of $\frac{5}{8}$ represents five parts of a whole which has been divided into 8 pieces. The denominator is the bottom number; it indicates how many parts in a whole. The numerator is the top number; it indicates how many of those parts are present.

To add or subtract: Multiply both the numerator and the denominator in each fraction by the same factor (number) to obtain a common denominator, then add (or subtract) the numerators.

- To multiply: Multiply numerator times numerator; denominator times denominator. A common denominator is not necessary.
- To divide: Take the inverse of the second term (reverse the numerator and denominator), then multiply.

- To simplify or reduce a fraction, divide both the numerator and the denominator by the same factor(s).

Rational Numbers: Any number that can be written as a fraction of whole numbers is called “Rational.” Some numbers cannot be written as a fraction of whole numbers; instead they form decimals that do not repeat in a consistent pattern. These are called irrational. A common example is pi.

Explanation

- [Types of Fractions](#) (Math Antics / YouTube)
- [Fractions: Comparing and Reducing](#) (GCFglobal.com)
- [Proper, Improper and Mixed Fractions](#) (ipracticemath.com)
- [Adding and Subtracting Fractions](#) (GCFglobal.com)

Practice

- [Equivalent Fractions](#) (iknowit.com)
- [Converting Improper Fractions and Mixed Numbers](#) (iknowit.com)
- [Adding and Subtracting Fractions with Unlike Denominators](#) (iknowit.com)

Decimals

Decimal values represent amounts between zero and one. A value may combine a whole number and a decimal such as 11.9 or -45.98. The most frequent application of decimals for many adults is money; think about the number of cents in a price such as \$5.49.

The decimal portion of these values may also be written as fractions. To convert from a fraction to a decimal, divide the numerator by the denominator.

- To add or subtract decimals, line the decimal points up, and add/subtract using standard steps. Bring the decimal point down in the same position.
- To multiply, ignore the decimal points, line up and multiply using standard long multiplication. Count the number of places to the right of the decimal point in the two original numbers. Count in from the right by that many places and insert the decimal point, e.g., 2.3×1.57 . Multiply as if it were $23 \times 157 = 3611$; count 3 places to move in and end up with 3.611.

Explanation

- [Introduction to Decimals](#) (GFCGlobal.org)
- [Converting Percentages, Decimals, and Fractions](#) (GFCGlobal.org)

Practice

- [Ordering Decimals to Thousandths](#) (iknowit.com)
- [Convert Between Percents, Fractions and Decimals](#) (mathgames.com)
- [Adding and subtracting decimals: word problems](#) (Khan Academy)
- [Word problems on multiplication and division of decimals](#) (Khan Academy)

Percents

Percents represent a ratio compared to 100. Values above 100% represent an increase above the initial amount. There are multiple methods to calculate, including converting the percent to a decimal (70% → .7) and multiplying, or cross multiplying a proportion: part:whole and %:100.

Explanation

- [Introduction to Percentages](#) (GFCGlobal.org)
- [Introduction to Percents](#) (mathisfun.com)
- [Percentages Made Easy!](#) (The Organic Chemistry Tutor / YouTube)

Practice

- [Finding Percents](#) (Khan Academy)
- [Percent of a Number: Tax, Discount, Tip](#) (mathgames.com)
- [Percent Word Problems](#) (Khan Academy)

Ratios & Proportions

A ratio is a comparison of two quantities related by multiplication/division. Daily life examples include staffing or student teacher ratios (12 students per teacher), or cooking examples (2 cups of water for 1 cup of rice.) Ratios may be written as a fraction, with a colon separating the values (12:1) or in words.

A proportion is two ratios which are equivalent to each other. To find a missing term in a proportion, cross multiply. Set the proportion up as two fractions, multiply the two values located diagonally across from each other, and divide by the remaining value. e.g., 5:10 is to 6:x → $5/10 \times 6/? \rightarrow 60/5 = 12$, therefore 5:10 is to 6:**12**.

Explanation

- [Ratios and Rates](#) (Math Antics/ YouTube)
- [Proportions](#) (Math Antics / YouTube)

Practice

- [Solving Proportions](#) (Khan Academy)
- [Proportional Relationships](#) (mathgames.com)

Objective 8: Understand basic concepts of algebra

Patterns, Tables & Rules

There are many possible number patterns. In math, arithmetic sequences refer to patterns formed by adding or subtracting a term to the last value, to generate the next. Geometric sequences are the result of multiplying or dividing.

Interest:

- [Pascal's Triangle](#) (TEDEd)
- [Nature by Numbers](#) (Cristobal Vila / YouTube)

Explanation

- [Generating Number Patterns](#) (helpingwithmath.com)
- [Common Number Patterns](#) (mathisfun.com)
- [Top 10 Secrets of Pascal's Triangle](#) (Math Hacks/Medium)
- [Factorial Function!](#) (mathisfun.com)

Practice

- [Increasing Growth Patterns](#) (mathgames.com)

Algebraic Symbols & Expressions

Variables stand for an unknown number, they could be any non-number symbol, but are generally letters (x, n etc.) Variables may be manipulated similarly to numbers; a variable with an exponent means that variable multiplied by itself etc. A coefficient, such as the 2 in $2x$, represents multiplication. $2x$ can be understood as 2 times the unknown value of x.

Simplifying an expression means writing it in a more concise form. Terms with the same variable, and the same power/exponent are considered "like", and may be combined (e.g., $3x^2 + 4x^2$ simplifies to $7x^2$)

Interest

- [Why all the letters in algebra \(Khan Academy\)](#)
- [Why X \(TED\)](#)

Explanation

- [Reading Algebraic Expressions](#) (GFCGlobal.org)
- [Variables and Expressions](#) (Super Easy Math / YouTube)
- [Simplifying Expressions](#) (GFCGlobal.org)

Practice

- [Writing Algebraic Expressions](#) (iknowit.com)
- [Evaluating Algebraic Expressions](#) (iknowit.com)
- [Evaluating an expression with one variable \(Khan Academy\)](#)
- [Combining Like Terms](#) (Khan Academy)

Functions

A function is a relationship or pattern. It is often compared to a machine; one number is put into the machine, something happens and another comes out. The first numbers are called inputs, and when graphed are represented on the x-axis. The second set are called outputs and are represented on the y axis of a graph.

For a pattern to qualify as a function, each input must yield one and only one output. On a graph this is tested with a vertical line: if any vertical line passes through a graphed line more than once it is not a function.

Functions are often written as $f(x) = 2x + 5$. This is read “f of x” or “the function of x” and may be evaluated by substituting input values for x.

- **Domain:** all the possible input values (the x value when graphed)
- **Range:** all the possible output values (the y value when graphed)

Explanation

- [Intro to Functions](#) (MathEase / YouTube)
- [What are Functions?](#) (Math Antics / YouTube)
- [Increasing and Decreasing Functions](#) (mathisfun.com)
- [Inverse Functions](#) (mathisfun.com)
- [Maxima and Minima of Functions](#) (mathisfun.com)
- [Piecewise Functions](#) (mathisfun.com)

Practice

- [Identify Functions](#) (Imathbitsnotesbook.com)
- [Evaluate Functions](#) (Khan Academy)
- [Domain and range of functions](#) (Khan Academy)

Linear Equations

In solving an equation (example: $16 = 2x + 4$), one is trying to determine the value of the unknown variable, the x. The key information is that the two sides of the equation are equivalent. For each operation, complete the opposite operation (addition and subtraction are opposites, multiplication and division are opposites) with the same number.

In the above example, the +4 is cancelled by subtracting 4. Complete the same math on both sides of the equals sign. In the above equation, add four to both sides, resulting in $20 = 2x$, then divide both sides by 2, yielding the answer: $10 = x$.

Equations with two variables, e.g., $y = 2x + 4$, cannot be ‘solved’ in this manner, but can be graphed on the coordinate plane, where they yield a straight line.

Explanation

- [What are Equations?](#) (helpingwithmath.com)
- [Solving Equations](#) (GFCGlobal.org)
- [Linear Functions](#) (mathantics.com / YouTube)
- [Algebra Mistakes](#) (mathisfun.com)

Practice

- [Basic Equations](#) (Iknowit.com)
- [Linear Equations and Functions](#) (Khan Academy)

Inequalities

- $A < B$ means A is less (smaller, a lower value) than B
- $C \leq D$ means C is less than or equal to D. If D were 5, C might be 3, 4.9999, -6 OR 5
- $E > F$ means E is greater (larger, a higher value than) F
- $G \geq H$ means G is greater than or equal to H. If H were 10, G might be 10, 10.965, 203, or 12

It may be helpful to remember that the larger (open) end of the $<>$ shape is towards the larger number, and the smaller (pointed) end is towards the smaller. (Many of us learned to imagine a hungry crocodile or alligator eating the larger number.)

When represented with an arrow on a number line, an open dot indicates greater/less than, a filled dot indicates greater than or equal to/less than or equal to.

To solve an inequality such as $3x - 2 > 7$, treat as if it were an equation UNLESS you multiply/divide by a negative number, in which case the symbol reverses (e.g., from $>$ to $<$)

Explanation

- [Solving Inequalities](#) (mathcenter.ac.uk)
- [Solving Inequalities](#) (mathisfun.com)

Practice

- [Graphing basic inequalities](#) (Khan Academy)
- [Solve Two-Step Linear Equalities](#) (mathgames.com)
- [Two-variable inequalities from their graphs](#) (Khan Academy)

Systems of Equations

A system of equations is a set of two or more equations that use the same variables. Solving a system means identifying the values of those variables that solve ALL of the equations. There are several different approaches, including graphing to identify points of intersection/overlap; substituting, by defining one variable in terms of the other; and elimination, when one variable can be cancelled by combining like terms.

Explanation

- [Beginners Guide to Systems of Equations](#) (Math Hacks / Medium)
- [Solving Simultaneous Equations](#) (helpingwithmath.com)
- [Solving Systems of Linear Equations: Substitution Methods](#) (MathEase / YouTube)

Practice

- [Is x,y a Solution to the System of Equations](#) (mathgames.com)
- [Systems of equations with graphing \(practice\)](#) (Khan Academy)
- [Solve a system of equations using substitution](#) (mathgames.com)

Quadratics & Parabolas

A quadratic expression is any expression that has a variable with an exponent of two. It may also have terms without exponents, but not with powers greater than 2. The standard form is $ax^2 + bx + c$.

When graphed on a coordinate plane, a quadratic function will yield a parabola (a smooth curve: imagine water from a fountain, or half of the McDonald's M symbol). A quadratic can be factored to yield two binomials (see, Polynomials, FOIL below.)

Interest

- [Parabolas](#) (RadioL/YouTube)

Explanation

- [Quadratic Equations](#) (mathisfun.com)
- [Factoring Quadratic](#) (mathisfun.com)

Practice

- [Interpret a quadratic graph](#) (Khan Academy)
- [Interactive Parabola](#) (mathwarehouse.com)
- [Factoring quadratics intro](#) (Khan Academy)

Polynomials

A polynomial is a mathematical expression containing several terms. A monomial has one term ($2x$), a binomial has two ($2x + 3$). Polynomials may have 2, 3, or more terms. If there are multiple terms, the standard form is to organize the terms with the largest powers/exponents first, and any additional terms in descending order.

FOIL is a mnemonic for multiplying two binomials, e.g., $(a + 2) \times (3a - 4)$. Distributive function calls for each term in the first binomial to be multiplied by each in the last. **FOIL** is First ($x \times 3x$), Outer ($a \times -4$), Inner ($2 \times 3a$), Last (2×-4). The resulting expression $3a^2 - 4a + 6a - 8$, may be simplified to $3a^2 + 2a - 8$.

Explanation

- [Definition of Polynomials](#) (mathisfun.com)
- [Simplifying Polynomials](#) (Math Antics / You Tube)
- [Starter Guide to Factoring Quadratics & Polynomials](#) (Math Hacks / Medium)
- [Multiplying Polynomials](#) (mathisfun.com)

Practice

- [Multiply monomials by polynomials: area model](#) (Khan Academy)
- [Multiply binomials by polynomials](#) (Khan Academy)

Objective 9 Understand geometry and measurement.

General Resources

- [Formulas](http://helpingwithmath.com) (helpingwithmath.com)

Units and Measurements

Metric measurements are based on powers of 10, and can be converted by multiplying or dividing by 10s, 100s, 1000s. They include centimeters, meters and kilometers for length (100 cm = 1 m, 1000m = 1 km), milliliters and liters for volume (1000 mL= 1L), and grams and kilograms for weight (1000 grams = 1kg) The prefixes indicate the scale, with centi- referring to hundreds, milli- to 1/1000 and kilo- to 1000).

Standard or Customary measures include inches and feet; (12 inches = 1 foot), ounces, pounds and tons (16 oz = 1 pound, 2000 pounds = 1 ton); fluid ounces, cups and gallons (8 fl oz = 1 cup; 64 fl oz = 1 gallon), and are less patterned than metrics, therefore each conversion factor must be learned individually.

Interest

- [Why the metric system matters](#) (TEDEd)
- [Why are there 5280 feet in a mile?](#) (Mental Floss / Pocket)

Explanation

- [Systems of Measurement | Weights and Measures](#) (skillsyouneed.com)
- [Metric System of Measurement](#) (mathisfun.com)
- [Conversion of Temperature](#) (mathisfun.com)

Practice

- [Compare and Convert Customary Units](#) (mathgames.com)
- [Length Conversions \(Standard Units\)](#) (iknowit.com)
- [Length Conversions \(Metric\)](#) (iknowit.com)
- [Which Customary Unit of Length is Appropriate?](#) (mathgames.com)

Shape Geometry: Area, Perimeter, Volume and Surface Area

- **Area:** The amount of space inside a two dimensional figure. Imagine: tiling a floor. For a rectangular shape: length x width. For other shapes, the formulas vary.
- **Perimeter:** The distance around the outside of a shape. Imagine: Fencing a yard. Add the length of all of the sides together.
- **Volume:** The amount of space inside a three dimensional figure. Imagine: packing a box. For rectangular prisms: length x width x height. For other shapes the formulas vary.
- **Surface area:** The amount of flat surface (area) on the outside of a 3D shape. Imagine: Wrapping a present. Find the area of each side and add together.

Explanation

- [Introduction to Geometry: Points, Lines, Planes and Dimensions](#) (skillsyouneed.com)
- [How to Calculate Area](#) (helpingwithmath.com)
- [How to Calculate Volume](#) (helpingwithmath.com)
- [Using Nets to Find Surface Area](#) (helpingwithmath.com)

Practice

- [Area of Squares and Rectangles](#) (mathgames.com)
- [Find Missing Side](#) When Given Area (Khan Academy)
- [Find the Volume of a Rectangular Prism](#) (Iknowit.com)
- [Area of Complex Figures](#) (mathgames.com)
- [Apply Volume of Solids](#) (Khan Academy)

Angle Relationships

Angles may be described as acute (less than 90 degrees, small and pointy), right (exactly 90 degrees, like the corner of a paper), obtuse (90-180 degrees, wide and open), straight (exactly 180 degrees, looks like a line), or reflex (more than 180 degrees.)

Opposite, or vertical, angles are formed by a pair of intersecting straight lines, such as an X. Opposite angles do not share a side; they meet only at the intersection (vertex) of the lines. They have the same angle.

Complementary angles add to 90 degrees. Supplementary add to 180. In a pair of complementary or supplementary angles, if one is known, the remaining value can be determined by subtracting from 90 or 180.

Transversal is a straight line that crosses a pair of parallel lines, creating two sets of four angles (8 total). It looks approximately like the not equals sign. (\neq) Because the two lines are parallel, the two sets of angles formed at each parallel are congruent (the same). That is, the top right angle in one set, is the same as the top right angle in the other.

Explanation

- [Properties of Angles](#) (helpingwithmath.com)
- [What are Acute, obtuse, right and straight angles?](#) (virtualnerd.com)
- [Angles Around a Point](#) (mathisfun.com)
- [What is a transversal?](#) (virtualnerd.com)
- [Parallel Lines, and Pairs of Angles](#) (mathisfun.com)

Practice

- [Complementary & Supplementary Angles](#) (Khan Academy)
- [Complementary, supplementary, vertical and adjacent angles](#) (mathgames.com)
- [Find Missing Angles in Triangles and Quadrilaterals](#) (mathgames.com)

Similar and Congruent Figures

Congruent figures are the same: size, angles, shape. They may be oriented differently (rotated etc.) but otherwise match exactly.

Similar figures have the same angles and general shape, but may not be the same size. Imagine one shape has been enlarged or shrunk on a photocopier.

Similar shapes are proportionate, so the side of one shape may be calculated using proportions (see Obj. 7) when the length of the corresponding side in a similar shape is known. For example, a rectangle with sides of 2 and 3 is similar to a rectangle with a short side of 4 and an unknown long side. A proportion $\frac{2}{3} = \frac{4}{x}$ can be cross multiplied to yield a side length of 6.

Explanation

- [What are similar figures?](http://virtualnerd.com) (virtualnerd.com)
- [What does congruent mean?](http://virtualnerd.com) (virtualnerd.com)
- [Congruent](http://mathisfun.com) (mathisfun.com)

Practice

- [Similar and Congruent Figures](http://mathgames.com) (mathgames.com)

Triangle Types and Angles

Triangles can be classified based on their sides and interior angles. No matter the shape, size, or type, all of the interior angles of a triangle add to 180 degrees. A missing angle may be found by subtracting the known angles from 180.

- **Equilateral Triangles:** All 3 sides are equal, all three angles are the same (60 degrees).
- **Isosceles Triangles:** Two sides are equal, two angles are equal. Imagine a simple drawing of a party hat. The two equal angles are located where the equal sides meet the third (unequal) side.
- **Scalene Triangles:** All three sides are different, all three angles are different.
- **Right Triangles:** A right triangle is any triangle that contains a right, or 90 degree angle. Its sides may also be isosceles (two are equal) or scalene (none are equal). See section on Pythagorean Theorem for more on right triangles.

Explanation

- [Triangle Angle Review](http://KhanAcademy.com) (Khan Academy)
- [Triangles](http://mathisfun.com) (mathisfun.com)

Practice

- [Types of Triangles](http://mathgames.com) (mathgames.com)
- [Find angles in triangles](http://KhanAcademy.com) (Khan Academy)

Pythagorean Theorem

Pythagorean Theorem: $a^2 + b^2 = c^2$ allows you to find the length of an unknown side in a right triangle. A right triangle has a right (90 degree) angle.

The hypotenuse is the longest side, opposite the right angle; it is always represented by c in the formula. Most commonly it is unknown and the lengths of the a and b sides are provided. Square both known sides, add together, and take the square root of that sum.

If the unknown side is one of the legs (shorter sides, connected to the right angle), reverse the formula: square the known sides, subtract, and take the square root.

There are a relatively small number of Pythagorean triples, combinations of sides where all are whole numbers. The most common are 3/4/5 or their multiples (6/8/10, 9/12/15 etc.)

Explanation

- [Football Video](#) (YouTube/ National Science Foundation)
- [The Pythagorean Theorem](#) (Math Antics / YouTube)
- [Pythagoras' Theorem](#) (mathisfun.com)
- [Pythagorean Triples](#) (mathisfun.com)

Practice

- [Pythagorean Theorem Intro](#) (Khan Academy)
- [Converse of the Pythagorean theorem](#) (mathgames.com)

Circles

- **Circumference:** The distance around the outside of a circle. When you draw a circle, your pencil likely traces the circumference. Circumference = π times diameter.
- **Diameter:** A measurement straight across a circle from one edge to the other, through the middle (imagine cutting a pizza into triangles) Diameter = circumference divided by π .
- **Radius:** The distance from the center of a circle to the edge. It is half of the diameter. Imagine the hands of a clock.
- **Pi (π)** Approximately 3.14. Pi is the ratio of the circumference to the diameter of any circle. It is irrational (the decimal numbers continue without repeating). It is programmed into many calculators, but for most purposes 3.14 is close enough.

Interest

- [The Infinite Life of Pi](#) (TEDEd)

Explanation

- [Circles, Circumference and Area](#) (Math Antics / YouTube)

Practice

- [Circles: Calculate Area, Radius, Circumference](#) (mathgames.com)
- [Area and Circumference of circles challenge](#) (Khan Academy)

Coordinate Geometry

The coordinate plane locates a point in two dimensions. Coordinates are written (2, 3) indicating the point is two units from the origin on the horizontal (x) axis, and three on the vertical (y) axis. (0,0) is the intersection of the two axes, it is referred to as the origin. Negative x values indicate points on the left side of the origin. Negative y values indicate a point below the origin.

The slope of a line is a measure of its slant or steepness. Positive slopes rise as they move right; negative slopes descend. To calculate a slope, find the ratio of the rise (change in y value/vertical) over run (change in x value/horizontal).

The intercept is the point where a line crosses the x or y axis. If “intercept” is used without an x/y indicator, it is assumed to refer to the y intercept.

Slope-intercept form: $y = mx + b$, where m = slope, and b = y intercept. A common format for linear equations.

Interest

- [Pixar: The Math Behind the movies](#) (TEDEd)

Explanation

- [Explore the Properties of a Straight Line Graph](#) (mathisfun.com)
- [Graphing on the Coordinate Plane](#) (Math Antics / YouTube)
- [Rate of Change & Slope of a Line](#) (helpingwithmath.com)

Practice

- [Graph points on the coordinate plane](#) (mathgames.com)
- [Translations Find the Coordinates](#) (mathgames.com)
- [Graph linear relationships word problems](#) (Khan Academy)
- [Graph from slope-intercept form](#) (Khan Academy)
- [Slope from a graph](#) (Khan Academy)

Objective 10: Understand data analysis, statistics, and probability

General

Interest

- [Inside OKCupid: The math of online dating](#) (TEDEd)

Charts, graphs & tables

Charts, graphs, and tables are all ways to represent data visually. Common forms include bar graphs, where vertical or horizontal bars of different sizes represent data points, or pie charts where a circle is divided into wedge shapes representing proportions of a whole. Less common but mathematically important include scatter plots, which resemble a coordinate plane, and histograms, which resemble bar graphs.

Interest

- [The Beauty of Data Visualization](#) (TEDEd)
- [Indexed \(thisisindexed.com\)](#)

Explanation

- [Graphs and Charts](#) (skillsyouneed.com)
- [Scatter Plots](#) (helpingwithmath.com)
- [Histograms](#) (mathisfun.com)

Practice

- [Choose the best graph type](#) (mathgames.com)
- [Practice: Read Histograms](#) (Khan Academy)
- [Scatter Plots](#) (mathgames.com)

Central Tendency

Central tendency is a set of simple statistics used to summarize a set of data.

Mean, commonly called average, is found by adding all of the values and dividing by the number of values. In the set 2, 3, 2, 4, 1 Calculate: $2 + 3 + 2 + 4 + 1 = 12 / 5 = 2.4$ (mean).

Median is the middle value, when arranged smallest to largest (1, 5, 7, 11, 13: median is 7.) If there are any even number of terms, there will not be a middle term. Find the mean of the two closest values. 2, 3, 2, 4, 1 -> 1, 2, 2, 3, 4 -> 2 (median).

Mode is the most frequently occurring. A given set of values may have multiple modes (more than one term repeats the same number of times.) A set without any repeating values does not have a mode. In the set 2, 3, 2, 4, 1; 2 is the mode.

Range is the distance between the highest and lowest values, it is found by subtracting them. In the set 2, 3, 2, 4, 1; $4 - 1 = 3$.

Explanation

- [Averages: Mean, Median and Mode](https://www.skillsyouneed.com/mean-median-mode.html) (skillsyouneed.com)

Practice

- [Mean, Median, Mode & Range](https://www.iknowit.com/mean-median-mode-range.html) (iknowit.com)
- [Interpret charts and graphs to find mean, median, mode, and range](https://www.mathgames.com/mean-median-mode-range) (mathgames.com)
- [Mean, median, mode, and range: find the missing number](https://www.mathgames.com/mean-median-mode-range-find-the-missing-number) (mathgames.com)

Statistics

Statistics is the study of data.

Frequency refers to the number of repetitions of a particular response. In a survey about cats vs. dogs, frequency might be the number of people who prefer cats. A two-way frequency compares two types of data, for example a chart with gender and cat/dog preferences, which counts men who like cats, women who prefer dogs etc.

Percentile arranges all results into 100 equal groups. It is commonly used to report test scores and similar data: a result in the 37th percentile means that score is better than 37% of all results.

The 25th percentile is also called the first quartile, the 75th is the 3rd quartile. The 50th is the median.

Explanation

- [Simple Statistical Analysis](https://www.skillsyouneed.com/statistics-simple.html) (skillsyouneed.com)
- [Percentiles – Introductory Statistics](https://www.youtube.com/watch?v=...) (Quantitative Specialists / YouTube)
- [Frequency Distribution](https://www.ipracticemath.com/frequency-distribution.html) (ipracticemath.com)

Practice

- [Create Frequency Tables](https://www.mathgames.com/create-frequency-tables) (mathgames.com)
- [Two-way tables, Venn diagrams, and probability](https://www.khanacademy.com/statistics-probability/a/mean-median-mode-range) (Khan Academy)

Data Analysis & Bias

Data may be biased in its collection or its presentation. A common collection distortion is a non-random sample, for example asking only users of a product about its quality. Common graph distortions include manipulation of the Y axis for bar or line graphs. Starting the axis at a non-zero factor can make small differences look large, using large units or small distances between units can make differences look smaller.

Explanation

- [Accuracy and Precision](https://www.mathisfun.com/accuracy-precision.html) (mathisfun.com)
- [Random Sample](https://www.mathisfun.com/random-sample.html) (mathisfun.com)

Practice

- [Improving Questions](#) (mathisfun.com)
- [Spotting Misleading Graphs](#) (TEDEd)
- [Identify representative, random, and biased samples](#) (mathgames.com)

Probability

Probability is the odds of an event happening. Assuming random and equally likely outcomes, theoretical probability is calculated as a ratio of the number of preferred outcomes to the number of total outcomes. For example the odds of rolling an even number on a standard die, is $3 / 6$. There are three even numbers on the die (2, 4, 6) out of 6 total sides.

Probability is often converted to a percent. 0% probability indicates an impossibility, 100% a certainty, 50% an outcome that is equally likely or unlikely, e.g., a coin toss.

The outcome of multiple events can be found by multiplying the probability of each event.

- **Independent probability:** the outcome of one event does not impact the outcome of subsequent events. e.g., flipping a coin.
- **Dependent probability:** the outcome of one event does impact subsequent events, e.g., dealing cards.

Explanation

- [What is Probability](#) (helpingwithmath.com)
- [The Basic Counting Principle](#) (mathisfun.com)
- [Probability: Types of Events](#) (mathisfun.com)
- [That Common Misconception about Probability](#) (Brett Berry / medium.com)

Practice

- [Intro to Theoretical Probability](#) (Khan Academy)
- [Probability of opposite and overlapping events](#) (mathgames.com)