



It's a Lot of Work!

How does taking a sample help in predicting?

Synopsis

This lesson extends the notion of rate, drawing attention to the capacity of ratios to help extrapolate from smaller amounts to larger ones. Students use ratios to predict how long it will take to perform a larger task based on the time it took to conduct a small portion of that task. They explore how the underlying ratio between the two quantities in the small sample guides estimation for larger quantities.

1. The class discusses ways to predict how long it will take to complete tasks at work or home.
2. In teams, students estimate how long it will take to accomplish three tasks based on the time it takes them to do a portion of each task.
3. After all estimates are displayed and methods discussed, the class agrees on a final estimated time it would take to complete all three tasks of the assignment.
4. The class lists advantages for performing the sample task more than once.

Objectives

- Conduct and describe a sample of work over a period of time
- Use the sample to make a prediction for a larger amount by reasoning with equal ratios

WHAT TO LOOK FOR IN LESSON 2	WHO STANDS OUT? (LIST STUDENTS' INITIALS)			NOTES FOR NEXT STEPS
	STRONG	ADEQUATE	NEEDS WORK	
<p>Concept Development</p> <ul style="list-style-type: none"> • Chooses a work sample and uses it to predict the amount of time for a larger amount of work • Maintains the relationship between the amount of work and the time taken 				
<p>Expressive Capacity</p> <ul style="list-style-type: none"> • Translates a work sample into a ratio in fractional form • Writes a ratio in several forms (a/b, $a:b$, a per b) • Can explain clearly whether two ratios are equal 				
<p>Use of Tools</p> <ul style="list-style-type: none"> • Uses pictures or objects to show that two ratios are or are not equal • Uses the property of equal fractions (multiplies both amounts by the same number) to create a ratio equal to a given ratio • Uses a table to keep track of the two amounts as they increase 				
<p>Background Knowledge</p> <ul style="list-style-type: none"> • Draws upon knowledge of piece work or sampling in workplace situations 				

Rationale

Estimating and making predictions from small tasks are integral to planning and accomplishing large tasks. In addition, businesses use sampling techniques to predict the quality of their products and services, as well as to estimate the quantities needed.

Math Background

The ratio of the quantity of work done to the amount of time required is assumed to be constant in the situations at the stations. In this second lesson, students keep the rate constant. Their pictures, tables, or the property of equal fractions should reflect this. As in *Lesson 1*, the central mathematical ideas of *Lesson 2* are as follows:

1. The multiplicative relationship between the amount of work done and the amount of time is always the same. In other words, the relationship *within* the ratio remains constant.

For example, when

$$\frac{1 \text{ potato}}{40 \text{ sec.}} = \frac{2 \text{ potatoes}}{80 \text{ sec.}} = \frac{3 \text{ potatoes}}{120 \text{ sec.}} = \frac{90 \text{ potatoes}}{3,600 \text{ sec.}},$$

the number of seconds is always forty times the number of potatoes.

2. An equivalent rate or ratio occurs when both amounts of the ratio are multiplied by the same number. Thus the relationship *between* any two equal ratios can be seen as a multiplication of both amounts by the same number to get from one to the other.

Rates often have to do with situations where the time varies with another variable, but all rates are not constant. When a person gets more experienced at a given task, his or her rate might speed up; or if the person is tired at the end of the day, his or her rate might decrease. Mathematically, situations where the rate changes do not lend themselves to proportional reasoning.

It is important to know that the term “sample” is not used in the statistical sense in this lesson, but in the everyday sense of sample—a portion, part, or segment.

Context

Just as in the first lesson, a familiar context is at the core of this activity. Time is money, especially for piece work. Although the tasks in this lesson are set in a volunteer context, students may have had jobs where they were paid by the unit of work completed or where they have been expected to complete a given number of tasks in a certain period of time.

Facilitation

This activity should be as student-focused as possible. Listen and allow teams of students to struggle among themselves to figure out how to address the challenges before them. Be sure to have calculators and manipulatives handy, but allow students to figure out for themselves the strategies they are going to use.

Encourage mental math.

Do not push for students to establish unit rate, especially if the numbers are unwieldy. As noted in *Lesson 2 in Action*, p. 35, students tried to come up with a unit rate (1 potato = 40 seconds or 0.667 minutes). This was uncomfortable for them, so they established a more comfortable ratio (3 potatoes = 2 minutes). Consider whether the unit rate is actually going to clarify the situation for students before suggesting that they use it.

Making the Lesson Easier

If students find that they have chosen unwieldy numbers in their sample, you may want to stop them before they get too far with building on their ratio. Suggest that they round their ratio so that the numbers are friendlier. For example, in one class, the students predicted that it would take 65 seconds to stuff 6 envelopes. Even though many teachers would suggest rounding 65 to 70, it made more sense for students to round to 60 (because 60 is 10 times 6). This rounding helps students deal more readily with the process of building to larger ratios, yet doesn't impact the integrity of the sample.

Making the Lesson Harder

Ask students to make predictions for a given amount of time (as opposed to a given number of envelopes or potatoes peeled): "So, if someone did this for two days, taking into account eight hours of time set aside to sleep, how many potatoes could he or she peel?"

Ask students to revisit their initial sampling rates to see how changing the time unit would affect the outcome. For example, if a group begins with peeling one potato in 40 seconds, what would that time be in minutes? ($\frac{2}{3}$ min.)

$$\frac{1 \text{ potato}}{\frac{2}{3} \text{ min.}} = \frac{2 \text{ potatoes}}{1 \frac{1}{3} \text{ min.}}$$

LESSON 2 IN ACTION

Students take various routes as they negotiate through the stations. The size of the sample and the way they build up from that sample to larger amounts is their choice, as is their selection of a second strategy. There is a lot of room to “play with the numbers,” and if they get muddled in the numbers, it is important to seek other strategies, such as creating a table. Here is one teacher’s description of how two women, Carla and Maya, worked together and of the teacher’s interventions.

According to their timed observation, it took 40 seconds to peel one potato. Maya grabbed a calculator to determine how many seconds it would take to peel 100, while Carla began her calculations using pencil and paper. They agreed that it would take 4,000 seconds.

I listened as they both tried different types of calculations. They divided 40 into 60 and got 1.5. They then tried [dividing] 60 into 40 and got 0.667. They didn’t like either number, but they thought 0.667 was particularly useless. They divided 100 into 20 and got 0.2, which they thought might give them the number of minutes left “to do the rest of the chores [stations].” They didn’t like 0.2; they felt it wasn’t the right answer.

After the students had made many false starts, I finally decided that we might try to make a chart up on the board. Because Carla had been developing a vertical chart, we continued with her format. We started building up seconds and potatoes, actually adding on 40 seconds for every new potato.

Potatoes	Seconds
1	40
2	80
3	120

When we got to 3 potatoes and 120 seconds (40×3), Carla realized that there were 2 minutes in 120 seconds. This was a nice breakthrough for the pair.

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Because I wanted them to see this in fraction form as well, I suggested setting up the ratio (fraction form) of 3 potatoes to 2 minutes, and we worked together to get to 12 potatoes in 8 minutes. Maya could readily see the pattern and started doubling each amount in her head. She suggested that the next ratio would be 24 to 16. Maya and Carla worked together, mostly doubling, but also combining.

Potatoes	Seconds	
1	40	} 2 min
2	80	
3	120	
4	160	} 2 min
5	200	
6	240	
7		
8		
9		

$\frac{3 \text{ pot}}{2 \text{ min}} = \frac{6 \text{ pot}}{4 \text{ min}} = \frac{12 \text{ pot}}{8 \text{ min}}$
$\frac{24 \text{ pot}}{16 \text{ min}} = \frac{48 \text{ pot}}{32 \text{ min}} = \frac{96 \text{ pot}}{64 \text{ min}}$
$\frac{99 \text{ potatoes}}{66 \text{ min}}$

Building this way, they were able to get to 99 potatoes in 66 minutes. They knew this was close to 100 potatoes, so I asked what would happen to the time if they added one more potato. They agreed that it would add another 40 seconds on to the 66 minutes, for a total of almost 67 minutes. I then asked how many hours 66 or 67 minutes would represent. Carla estimated that it would take “1 hour and 6 minutes.”

I would say that my role as a teacher was to encourage the two women to stay as close to the real situation as they could, visualizing the balance between potatoes and minutes. Their scramble for numbers on the calculator seemed to disconnect them from the meaning.

*Donna Curry
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LESSON 2



It's a Lot of Work!

How does taking a sample help in predicting?

We often take a sample—a portion or piece—to understand something about the bigger picture. For example, to find out what percent of American households have dogs, you could poll a small number of people and then use proportional reasoning to estimate how many dogs there are in the total number of U.S. households. In this lesson, you will conduct a **sample** of work to predict. Use the idea of keeping ratios equal to predict for larger amounts.



Activity: How Long Will It Take?

You offered to help at a community event and said you were willing to volunteer about four or five hours of your time. The coordinator took you up on your offer and assigned you the following jobs:

- Stuffing 1,000 flyers
- Peeling 50 pounds of potatoes for a huge potato salad
- Rolling 10,000 pennies from the penny-toss

You think these jobs are going to take a lot more time than the four or five hours you offered, but you would like to get a better idea of exactly how much time these jobs will take.

To get a good estimate of how much time these jobs require, you conduct a sample of each activity and use the property of equal fractions to figure out the time needed for the total job. How long do you estimate the assignment will take?

Work with your team to answer the questions for the activity at each station.

- Take a sample of how long it takes to do a part of the job.
- Use the sample rate to predict the time needed to complete the whole job. Show your reasoning.
- Be sure that everyone on your team can explain how the prediction was made. (This might require more than one strategy to explain the team's reasoning.)

Summary

1.

Tasks	Our Time Estimate
Folding and stuffing 1,000 envelopes	
Peeling 50 lbs. of potatoes	
Rolling 10,000 pennies	
Total Time	

2. Compare your time estimates with the time estimates from other teams.

What would you say is a good estimate of time? Why?



Practice: Time to Be Proportional

Time is proportional. There are 60 seconds in one minute, every 60 minutes is one hour, and every 24 hours is one day. See how quickly you can figure out the missing numbers in these time-related ratios. Do the math in your head, using what you know about the rule of equal fractions.

1. $\frac{24 \text{ hours}}{1 \text{ day}} = \frac{\quad}{3 \text{ days}}$

2. $\frac{60 \text{ minutes}}{1 \text{ hours}} = \frac{180 \text{ minutes}}{\quad}$

3. $\frac{48 \text{ hours}}{\text{days}} = \frac{72 \text{ hours}}{\text{days}}$

4. $\frac{120 \text{ seconds}}{\text{minutes}} = \frac{3,600 \text{ seconds}}{\text{minutes}}$

5. $\frac{2 \text{ minutes}}{\text{seconds}} = \frac{4 \text{ minutes}}{\quad}$

6. $\frac{12 \text{ hours}}{\text{days}} = \frac{\quad}{2 \text{ days}}$

7. Choose one of the proportions above and use a diagram to check your answer.



Practice: Wasting Water

How much water does a dripping faucet waste in a year? That is an important question as we become more and more concerned about limited resources such as clean water.



Try an experiment: Take three samples of faucets dripping in your own home. Predict how much water would drip from those three faucets together in a year. If you do not have dripping faucets, turn your faucet on to drip a bit for the purposes of the experiment.

1. How I chose my samples:

2. Drip samples:
Sample 1: _____
Sample 2: _____
Sample 3: _____
3. Explain your answer and the way that you figured it out.

Think!

There are 60 minutes in an hour. How many minutes are there in a day?
How many days in a year?



Practice: Typing Tests

Over time, some people get better at skills while others do not. Below are four consecutive monthly typing test results for Kayla.

Month	Words Typed	Minutes	Minutes per Page (250 words)
January	100	5	
February	125	5	
March	200	8	
April	100	2	

1. Complete the last column in the chart above. Use the results for each test to predict how long it would have taken Kayla to type a page at this rate.
2. Tell the story of what happened to Kayla's typing skills over time. Include the number of words per minute during the time when she was the slowest. Also tell how many words per minute she typed at her fastest speed.
3. Use a diagram or the property of equal fractions to show whether Kayla's typing speed is the same for any two months.



Practice: Who Is the Fastest?

Several bikers have been chatting about their best race times. Each has won a race, although they have never competed against one another.

Each is claiming that he is the fastest biker. Look at their best times and then determine which of the racers is actually the fastest.

Biker	Race Length (in miles)	Best Time
Drake	50	2 hr. 30 min.
Leon	45	2 hr.
Denton	100	4 hr.

1. List the bikers from slowest to fastest. Give each of their speeds in miles per hour.

2. Show by diagram or the property of equal fractions why their rates are not the same.



Test Practice

- It took Jai 30 minutes to roll 100 of his 1,000 newspapers. At this rate, how long would it take him to roll all of them?
 - 5 hours
 - 30 hours
 - 50 minutes
 - 100 minutes
 - 3,000 minutes
- A stamping machine can stamp 500 letters in 3 minutes. At this rate, how long would it take the machine to stamp 10,000 letters?
 - 6 minutes
 - 50 minutes
 - 60 minutes
 - 1,500 minutes
 - 30,000 minutes
- It took Marie 30 minutes to paint faces on 4 jack-o-lanterns. Which rate below is the same as Marie's?
 - 2 hours to paint 12 faces
 - 4 hours to paint 30 faces
 - 5 hours to paint 20 faces
 - 75 minutes to paint 10 faces
 - 400 minutes to paint 30 faces
- A printing machine can print 250 business cards in 2 minutes. At this rate, how many cards can it print in 5 minutes?
 - 100
 - 500
 - 625
 - 1,250
 - 2,500
- A bread slicer can slice 50 loaves of bread in 10 minutes. At this rate, how many loaves can it slice in 2 minutes?
 - 2
 - 5
 - 10
 - 25
 - 100
- Johan lays bricks for patios. On average, he can lay 50 bricks in an hour. At this rate, how many hours would it take it to lay a patio that will need 275 bricks?