

Dear ABE teacher:

This packet is designed for use in families with children in upper elementary and middle school.

- The materials are designed to be “low floor—high ceiling.”
- The materials take a “making” approach to math learning. That means students will engage in math by designing and constructing objects. Math topics, spanning grades 4 – 7, include measurement and circle concepts.
- Throughout, we include the *why* as well as the *how*.

**Math in this packet:**

- Measurement of length in problem solving
- Parts of a circle—radius, diameter, circumference—and definitions, measuring, and relationships
- $\pi$  as the constant that relates the diameter and circumference



The Swan, by Hilma af Klint

Contact us at [MPACT@terc.edu](mailto:MPACT@terc.edu) for more information about our packets for teaching and learning at home, as well as our in-class program that will begin again when students come back to the school buildings.

# Making a Toy with Wheels for a Younger Child from Stuff Around Your Home

For caregivers:

- Are you stressed due to school closures?
- Are your kids bored?
- Not sure how math could help?

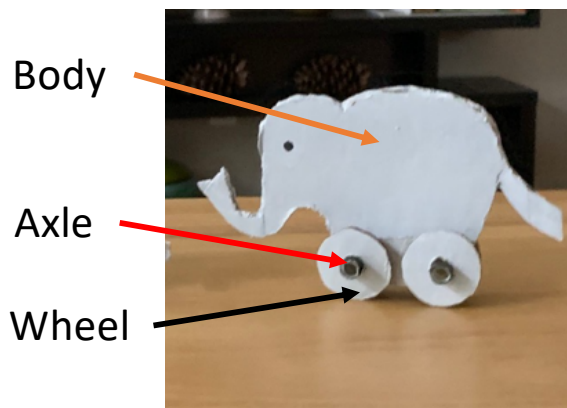
This packet, designed for families with children in grades 4-7, includes activities for making and math, for you and your kids to do together or for your kids to do on their own. You don't have to know the math in advance.

Be open to exploration, and ask questions such as

- "How did you do that?"
- "How do you know?"

## A sample toy

You can make many different designs.



## Materials

For body and wheels



Corrugated cardboard—thick cardboard with layers. Also, thinner cardboard such as a cereal box.

For axles



Drinking straws

For attaching parts together



Strong tape—packing tape, electrical tape, or masking tape

Also these tools:

- Scissors or serrated knife (use only with a grown-up, found in the kitchen)
- Pencils or pens
- Ruler marked in centimeters (cm)
- String
- Optional: paint and markers

You can use other materials if you cannot find these. Instead of drinking straws, you can use rolled up paper tubes. Instead of tape, you can use glue and paper.

## Step 1: Collect ideas

- Talk to a young child to find out:  
Do they like toy animals? Cars?  
Trains? What else?  
What are their favorite colors?



### Pay Attention to the Toy Requirements

- The body can be narrow or wide.
- The wheels must support the toy and roll so the toy can move.
- The axle must be long enough to go through the body and the wheels, with room for tape on both sides.

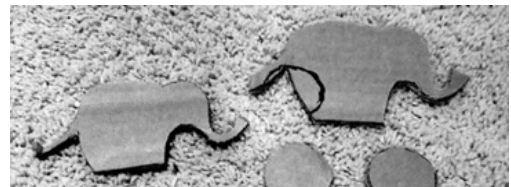
## Step 2: Design on paper

- Use scrap paper.
- Sketch your ideas for the child's toy.
- Choose the idea you want to use.



## Step 3: Draw and make parts

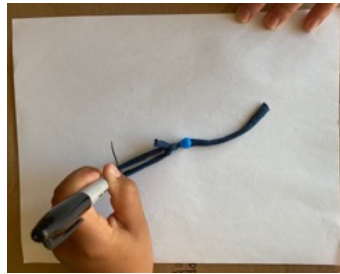
- Draw everything *except* the wheels on cardboard exactly as you want them to be.
- Draw the toy parts exactly—the right size and shape to cut out.
- Cut them out.
- It can be difficult to cut cardboard. Keep at it!



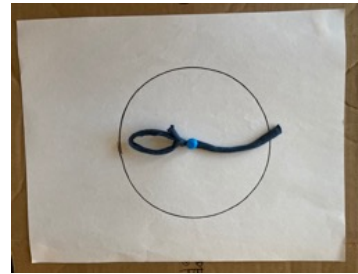
## Before you make the wheels, learn how to make a circle



- Cut ribbon a little bit longer than the radius you want for your circle.
- Make a loop for your pencil or marker.

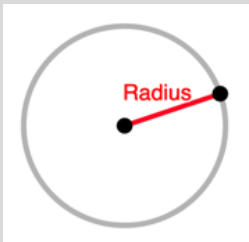


- Use a pushpin to attach the ribbon to the pad of paper, where you want the center of the circle. You can also use your finger.

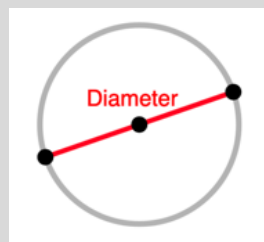


- Keep the string tight and draw all the way around to make the circle.

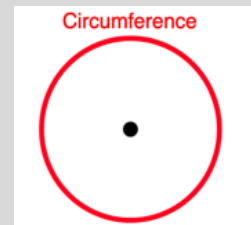
The **radius** of a circle is a line segment that goes from the center of the circle to the outer edge.



The **diameter** is a line segment that goes from one side of the circle to the other, passing through the center.



The **circumference** of a circle is the distance all the way around the circle. You trace the circumference each time you make a circle with your string and pencils.



- Draw three wheels of different sizes on paper. Keep these papers to use later.
- Talk about it: How does this way of making circles use each of these parts of a circle (above)?

### Step 4: Draw and make the wheels

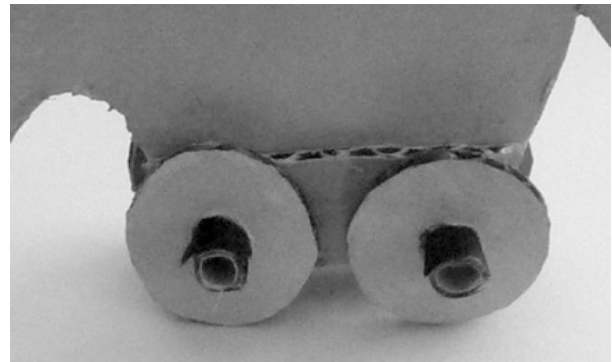
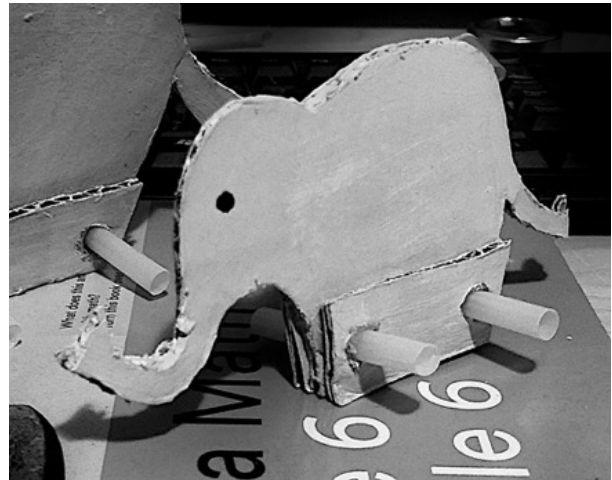
The wheels must support the toy and roll so the toy can move.

- Choose the wheel size you want to use, draw four of them onto cardboard, and cut them all out.



## Step 5: Put the toy together

- Make the body. If needed, place cardboard strips along the bottom to make a good base for the wheels.
- Make the axles long enough to go through the toy and the wheels, with a little bit extra on each end.
- Attach the two axles. Poke holes in the body (with scissors) and push the axles through or tape the axles to the bottom.
- Poke holes in the center of the wheels. The holes must be large enough to put the axles through and let the wheel turn.
- Put the axles through the holes in the wheels.
- Put tape on the end to hold the wheels in place. Make sure the wheels will still turn.



## Step 6: Try out your toy

- See if your toy rolls, on different surfaces
- If needed, change the toy so it stays up and rolls.

## All about Circles

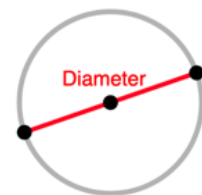
- Use the paper wheels (circles) you made. Draw lines on each circle to show the *radius* and the *diameter*.
- Measure the length of the *diameter* and *radius* for each of the three wheel sizes you made. Be sure you go through the center. Use centimeters if possible.

Circle	Radius	Diameter
Example	1 cm	2 cm
Small		
Medium		
Large		

We did the first one for you.

- What relationship do you see between the length of the *radius* and the *diameter*?
- Is that relationship true for any circle?
- Talk about it: Why do you think so?
- Use a string to measure the length of the *circumference* of each of your circles
- Fill in the table:

The **diameter** is a line segment that goes from one side of the circle to the other, passing through the center.



Circle	Circumference	Circumference ÷ Diameter
Example	6.4 cm	3.2
Small		
Medium		
Large		

- Predict: A circle has diameter 10 cm. What do you think the circumference is?
- Predict: What could be a rule for finding the circumference if you know the diameter?



## Even more about circles: introducing $\pi$

There is a special number that always relates the circumference of a circle and its diameter. You estimated it on page 5. The number is  $\pi$  (say “pi” with a long i).

□ Fill in C (for circumference) and d (for diameter) in the following formula:

$$\underline{\hspace{2cm}} = \pi \bullet \underline{\hspace{2cm}}$$

$\pi$  is an *irrational number*. That means that its digits to the right of the decimal point *never* form a repeating pattern. We can use estimates of  $\pi$  when finding the circumference and area of a circle.

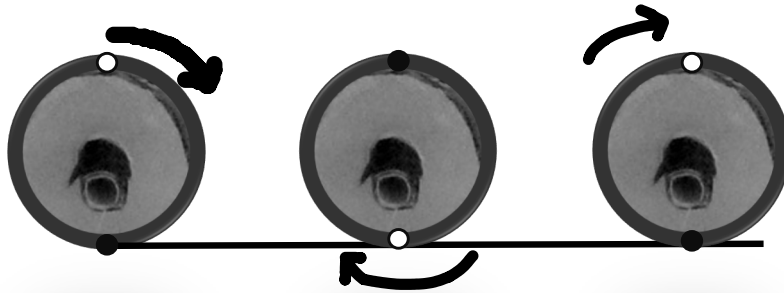
Two common estimates for  $\pi$  are **3.14** and **22/7**. But neither is *exactly*  $\pi$ .

3.141592653589793238462643383279502  
88419716939937510582097494459230781  
64062862089986280348253421170679 ...

*$\pi$  with many digits. Notice how they do not form a repeating pattern. (How can you check?)  
There are **infinitely many more** digits of  $\pi$ .*



## The parts of a circle in problem solving

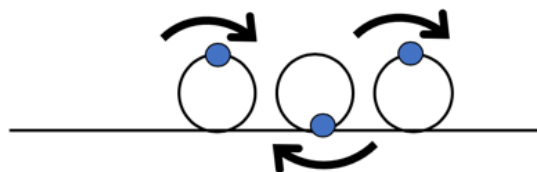
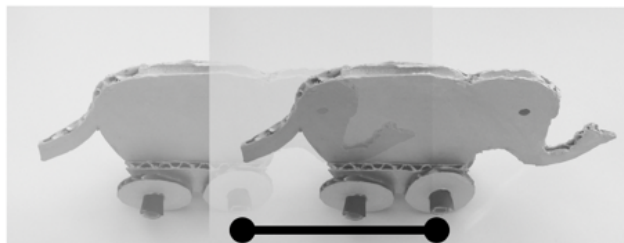


This picture show one full rotation of a wheel as it rolls over a rug on the floor.

1. Predict: When the wheel made one full rotation, how far would it roll?
2. Try it with your toy. Make a mark at the top of your wheel and roll it until that mark comes up on top again.
3. Were you right? Roll it along a ruler to see how far it went.
4. Explain: how do your measurements relate to circumference of the circle?
5. Apply: If the wheel makes three full rotations, how far will your toy go? Check with your toy.

## Problem solving

6. What if you make a new toy that is larger than your original toy. The radius of the wheels for the new toy is twice as long as the wheel of the original toy. Figure out how far your new toy will go with one full rotation of its wheels. Think about how you answered question #1, to help you.
7. You want to make a toy with wheels that can go about 15 cm with one full rotation of the wheels. Figure out the size of the wheels you should make.





## Answers

p. 3 How does this way of making circles use each of these parts of a circle?

The radius is the length of the string from the center to the location of your pencil when drawing a circle. Students move the radius around the paper to make the circle. The circumference is how far your pencil travels from start to end to draw a circle.

p. 5

Answers in the table will vary, depending on the size of wheels students make. What relationship do you see between the length of the *radius* and the *diameter*?

*The diameter is 2 times the radius.*

Is that true for any circle? Yes Talk about it: Why do you think so?

*Because the diameter goes through the center, it is made up of two radii that start in the center. These radii are the same length. So a diameter has to be twice as long as a radius. This will be true no matter what size the circle is, because you can draw a diameter and two radii that make it up, no matter how large or small your circle is.*

Answers in the table will vary, depending on the size of wheels students make.

Predict: A circle has diameter 10 cm. What do you think the circumference is?

*The circumference is about 30 cm or something close.*

Predict: What could be a rule for finding the circumference if you know the diameter?

The circumference is approximately 3 times the diameter.

This relationship is true for any circle. There is nothing special about the small, medium, and large circles you measure. We could have picked any circle to measure. So the relationship is likely true for any circle.

p. 6

$$C = \pi d$$

p. 7

Q1. Some kids might give a prediction by estimating how far it could roll by mentally visualizing the path.

Q4. Sample answers include: my measurements are close to the circumference of the wheel. The distance of the path the wheel rolls match with the circumference of the wheel.

Q5. Answers vary depending on the wheel size. Because the wheel makes three full rotations, the distance it rolls should be three times the circumference of the wheel.

Q6. If the radius of the new wheel is twice as long as the radius of an original toy, the distance the new wheel goes with one full rotation would be twice as long as the distance that the original wheel goes with one full rotation, or the circumference of the original wheel.

Q7. Students learn from the previous problems that with one full rotation, a wheel goes as the same distance as the circumference. That means, 15 cm is the circumference of the wheel. From the circumference, students can figure out the diameter of the wheel by dividing 15 by  $\pi$  and get approximately 5 cm.

