TEACHING ALGEBRAIC NOTATION

by Melissa Braaten

Helping students develop meaningful fluency in algebraic notation can help them apply their reasoning skills and can decrease "algebra anxiety," as the symbols and grammar start to become meaningful and familiar. This three-part resource reviews the meaning of notation in algebra, discusses some common student misunderstandings, and offers teaching strategies for promoting lasting student comprehension.

PART 1: STARTING OFF: DIFFERENT WAYS OF WRITING OPERATIONS

Struggling with notation can be a barrier to students with otherwise good reasoning skills. Gaining fluency and accuracy with notation is a process that takes a lot of time, practice, and cyclical review. I have found the most effective way to help students gain this fluency is to embed notation instruction and practice alongside reasoning and problem-solving activities from the very beginning. In this document, we will look at some aspects of algebraic notation that are important for students to learn, why it can be confusing, and some suggestions of math routines and teaching techniques that can help students improve and eventually master the symbols, syntax, and punctuation of algebraic equations. Part 1 will start off with alternate ways of writing multiplication and division that become more common as students get into algebra and intermediate-level mathematics.

Most intermediate-level math learners are familiar with the arithmetic symbols for multiplication (×) and division (÷). However, as they start to move into algebra, other symbols for these operations become more common. My first focus of notation instruction when I begin my algebra unit (a full year of curriculum at the intermediate level) is helping students understand alternate ways of writing these operations, which they will encounter more frequently as they study algebra and other secondary-level math.

Multiplication

Students need to learn how to write multiplication without the use of the × symbol. Since *x* is a popular variable in algebra, it can be very confusing to also use × for multiplication.

$$x \times 4 = y$$

While this representation is technically correct, it is not what students will see in math textbooks and standardized tests.

For constant numbers, parentheses around one or more factors indicate multiplication. For example,

$\mathbf{4(5)}$ **has the same meaning as** $\mathbf{4 \times 5}$

$\mathbf{3(4)(5)}$ **has the same meaning as** $\mathbf{3 \times 4 \times 5}$

Why it's confusing

Arithmetic symbols for operations are always found in between the numbers they are operating on.

$$4 \times 5$$

However, parentheses surround one of the factors, and so appear on both sides. Sometimes, parentheses are used to contain operations AND to indicate multiplication, as in this example:

$$4(3 + 1)$$

In this case, the second parenthesis is even further from the point where the operation is occurring, and the identity of the second factor (3 + 1) is not immediately obvious.

Because of the commutative property of multiplication, factors can be reversed without changing the product.

$$4 \times 6 = 6 \times 4$$

When written with parentheses, this can be harder for students to recognize, since the parentheses around only one of the factors makes it seem as if they are more "different" from each other.

$$4(6) = 6(4)$$

For variables, we don't use any symbols for multiplication. Instead, the constant is written next to the variable

$5a$ **means "five times** a**"**

or the variables are written next to each other

ab **means "**a **times** b**"**

Why it's confusing

First of all, there is no symbol for multiplication anymore! Students have to infer multiplication from the placement of the constants and variables.

I discourage the use of parentheses around variables for multiplication

$$5(a)$$

although this is natural for students to do at first. However, later on parentheses around variables are used in function notation $f(x)$, so I work hard to emphasize that they are not used in variable multiplication.

Also, while multiplication factors can be reversed

$$4(6) = 6(4)$$

$$ab = ba$$

when multiplying a constant by a variable, the constant is conventionally written first

$$5a\text{, rather than } a5$$

This is done to avoid confusion with exponents ($a5$ looks a lot like a^5).

Division

As students move into algebra, they will encounter the division bar or fraction bar to indicate division.

$$10 \div 2 \text{ becomes } \frac{10}{2}$$

$$(a + 2) \div 5 \text{ becomes } \frac{a+2}{5}$$

Why it's confusing

It looks like a fraction. (Part-whole fractions and part-part ratios can be equivalently thought of as quotients, but this is not usually well understood by intermediate level learners).

The length of the division bar functions as a type of "parentheses" without having to actually write parentheses. The expression on top of the division bar is the dividend, and the expression on the bottom is the divisor. Therefore, it is important that students think about how far the bar should extend when writing their expressions.

For example, in the problem $\frac{a+2}{5}$ the dividend is $a + 2$, not just a.

Teaching tips

Incorrect, technically correct, and preferred

When I am giving student feedback, I distinguish between expressions that are incorrect, technically correct, and preferred.

For example, a student wants to write "variable *x* times 2."

If the student writes x^2, that is incorrect. The notation they chose has a different meaning.

If a student writes $x \times 2$, or $x2$, I will explain that this is correct, however, they will not usually see it written this way. I would then point them to the preferred notation, $2x$. I would also repeat the reason why this notation is preferred: to avoid confusing *x* (variable) with × (multiplication operation), or to avoid confusion with exponents. I want them to know that preferred notation is not arbitrary.

Connect symbols to vocabulary and concepts

Ideally, students should connect new notation with vocabulary and concepts that they already understand, so this might require some teaching and backfilling. For example, being able to refer to a "factor" as a number being multiplied is helpful when trying to help students identify the factors in

$$\mathbf{4a} \qquad \textbf{or} \qquad \mathbf{bc} \qquad \textbf{or} \qquad \mathbf{4(3+1)}$$

In addition, the concept of a factor also involved understanding the commutative property of factors, which can help students understand that "*a* times 4" can be written as $4a$.

For division, I teach students the meaning of dividend and divisor and how to identify them in different common division notations. Part of the concept of a dividend and a divisor is that they have distinct roles in the operation and are NOT interchangeable.

This vocabulary is helpful when trying to help a student distinguish between expressions like $\frac{a}{2} - 1$ and $\frac{a-1}{2}$. We can ask the question, "What is the dividend?" which is a more conceptual question than "which operation comes first?"

Some math routines for practice

After explicitly teaching the needed notation (and as needed, the accompanying vocabulary and concepts), students need a lot of exposure and practice to develop fluency. Here are some of my favorite routines that incorporate this practice.

Number of the Day

With number of the day, students write expressions equal to a given number. You can add various other constraints, such as requiring them to use multiplication and or division in their expressions. I collect a sampling of student expressions on the board to discuss. If students use arithmetic notation, I acknowledge it as correct and use it as an opportunity to have the class practice rewriting it in algebra notation. As students become more comfortable with the routine, I challenge them to create more complex expressions.

Expand and Simplify

I heard about this routine from a [Landmark School]() training many years ago. You have students start with a number, say, 15. They write it down. Then they expand it, one step at a time. For example, a student might write

$$15$$

$$10 + 5$$

$$2(5) + 5$$

$$2(2+3) + 5$$

After they expand the expression a certain number of times, they go back and simplify it (step by step) back to the original number.

I do lots of variations of this routine. Sometimes I have students expand, then switch with a partner and simplify their partners expression. Having to communicate with a partner through an expression means they have to use notation accurately!

Translating words to equations in both directions:

As students are learning to write algebraic rules and functions, they are asked to write them both in words (using the structure, take *x*, _______, get *y*), then to translate that into an equation. I give them frequent practice “translating” in both directions, from words to equations and from equations to words.

No matter what routines you use for practice, frequent exposure and review is the most effective. Warm up routines, homework problems, quizzes; keep exposing your students to these notations and informally assessing how accurate and fluent they are with them. Don’t assume they’ve mastered it until they are interpreting and writing the new notation correctly. And even then, an occasional review can help cement the knowledge further!

PART 2: READING OUTSIDE-IN: PARENTHESES AND THE STRUCTURE OF EXPRESSIONS

Struggling with mathematical expressions can be a barrier to students with otherwise good reasoning skills. We are socialized by reading to process information left to right, but this does not always serve students well when they are trying to interpret numerical and algebraic expressions. In this section, we will look at ways to help students identify the structure of numerical and algebraic expressions, so they can proceed to work with those expressions accurately (and confidently!)

A big shift occurs when students transition from beginning to intermediate-level math: expressions and equations that were straightforward and mainly interpreted left to right

$$4 \times 5 = 20$$

$$4 \times 5 + 2$$

start to become more complex:

$$3(6 + 1)$$

$$\frac{3(7 + 7)}{2}$$

$$3\left[\frac{12 + 4}{8} + \frac{2(35)}{12 + 2}\right]$$

At this point, some students become overwhelmed or intimidated, unsure where to start. Let’s break down what is happening in a complex expression like this, and how we can teach students to navigate it.

The role of order of operations

Order of operations (think “PEMDAS”, “Please Excuse My Dear Aunt Sally” or another mnemonic) is often taught as an algorithm, or a list of things to do in a certain order.

1. Parentheses (Do operations in parentheses)
2. Exponents/Roots
3. Multiplication
4. Division
5. Addition
6. Subtraction

Students are then given multistep expressions (sometimes with only arithmetic notation) and told to follow the order of operations.

This traditional way of teaching order of operations can backfire in a couple of ways.

Inverse operations

Multiplication does not have to come before division. As inverse operations, multiplication and division are two sides of the same relationship, and one can be transformed into the other. For example:

$$4 \times 24 \div 3$$

This can be solved from left to right, by multiplying 4 × 24 = 96, then dividing 96 ÷ 3 = 32. However, it can also be solved by dividing first, 24 ÷ 3 = 8, then multiplying 8 by 4 = 32.

The traditional way of going through a horizontal expression left to right and solving step by step using PEMDAS does not suggest this (possibly easier) approach.

Why does this work? The important idea with division is to make sure each number is performing its proper role as part of the dividend or the divisor. Algebra notation can make this a little bit easier to see

$$\frac{4(24)}{3}$$

Since 24 is a factor in the dividend and 3 is the divisor in both approaches, they will lead to the same answer.

We can also demonstrate this by transforming division into multiplication.

$$4 \times 24 \div 3 = 4 \times 24 \times \frac{1}{3}$$

Because multiplication is commutative and associative, we can multiply the three factors in any order we want.

Addition and subtraction are also inverse operations, and addition does not have to be done before subtraction. For example,

$$5 + 34 - 11$$

This could be solved by adding 5 + 34 = 39, then subtracting 39 – 11 = 28.

Or, this could be solved by first subtracting: 34 – 11 = 23, then adding 23 + 5 = 28.

Why does this work? In either method, 11 is the subtrahend (the “minus” number). The other quantities are part of the minuend (the total or starting amount).

Another way to make sense of this is to transform subtraction into addition.

5 + 34 + (-11)

Because addition is commutative and associative, these can be added in any order.

A misunderstanding of PEMDAS can lead a student to calculate incorrectly, in addition to missing opportunities to make the operations easier. For example, if a student thinks that addition MUST come before subtraction, they might incorrectly simplify 14 – 3 + 5

by adding 3 + 5 = 8, then subtracting 14 – 8 = 6. However, 5 is not part of the subtrahend (in other words, 5 should not be part of what is taken away). The correct interpretation subtracts a 3 from the total, then adds on a 5.

14 + (-3) + 5, simplified in any order is equal to 16.

Parentheses

Parentheses can be incredibly confusing for students. For one thing, as students enter intermediate mathematics they start to see parentheses used for multiplication:

4 × 5 becomes 4(5) [refer back to Part 1 for a more in-depth discussion]. Parentheses used for multiplication are not what the P in PEMDAS is referring to.

Parentheses can also be used to "group" operations. I prefer to use "grouping" rather than "do this first" because some expressions contain many parentheses, and some have nested parentheses (parentheses inside parentheses). So, what is supposed to be done first?

If we look at

25 – (4 + 5)

the parentheses are grouping 4 + 5. In this case, it is not 4 that is being subtracted from 25, but the result (sum) of 4 + 5. This is where precise math vocabulary can come in very handy

25 – (4 + 5) THIS (indicating the entire parentheses) is the subtrahend.

4(9 + 2) THIS (indicating the entire parentheses) is the second factor.

Students need to see parentheses as referring to the value that is the result of the operations inside of it. In the example above, (9 + 2) is referring the value of the sum, which is 11, and that value (11) will then play another role in the expression (as a factor, multiplied by 4).

When helping students interpret expressions on the board, I usually highlight or circle parentheses. They are a math symbol with two parts, which is new for students coming from arithmetic. They have to pay attention to both sides, since it "groups" everything in between.

One last note about parentheses and PEMDAS: sometimes we can't simplify parentheses first. Think about an algebraic expression like:

$$4\,(a + 2) + 6a$$

Since we don't have a value for *a*, we can't simplify *a* + 2. However, the parentheses show that *a* + 2 is a factor multiplied by 4. Because of the distributive property, I can break up that factor and multiply 4*a* and 4(2).

Then I can simplify:

$$4\,(a + 2) + 6a$$

$$4a + 4(2) + 6\text{a}$$

$$10a + 8$$

Order of Operations without PEMDAS

We can teach students how to correctly interpret and solve complex expressions (like the one in the introduction) without leaning on PEMDAS as an algorithm. Instead, students need to understand:

- Inverse operations (multiplication/division and addition/subtraction and how they are related)
- Algebraic notation for multiplication and division (see Part 1)
- It's also helpful to know precise vocabulary like *addend*, *sum*, *factor*, *product*, *dividend*, *divisor*, *quotient*, *minuend*, *subtrahend*, and *difference.* (I start this vocabulary with beginning students and simple one-step operations)

Then we can teach two additional concepts:

1. Parentheses can be used to group operations and refer to their result as a single value.
2. Multiplication and division come before addition and subtraction.

Let's see how these ideas could be used to help students accurately (and in many cases, flexibly) simply a complex equation.

$$4\,(5 + 2) + \frac{10}{2} - 3(4)$$

In this expression, I would want students to notice that (5 + 2), parentheses are grouping 5 + 2 and that the sum of 5 + 2 is the factor that will multiply by 4.

$$\mathbf{4(7) + \frac{10}{2} - 3(4)}$$

They then need to understand that the multiplication and division must be completed before they can do the final addition and subtraction. The multiplication and division can be done in any order.

$$\mathbf{28 + 5 - 12}$$

The addition and subtraction can be performed in any order, as long as 12 remains a subtrahend (or negative).

$$\mathbf{21}$$

Reading Outside In: Looking for Structure

Traditional approaches to PEMDAS typically teaches students to look for what they need to simplify first. Instead, I encourage students to start by looking for the "big picture". In other words, what happens last?

Here is our expression from before:

$$\mathbf{3\left[\frac{12+4}{8} + \frac{2(35)}{12+2}\right]}$$

Let's work it Outside-In.

What is the big picture? In other words, what is the last thing that is going to be done?

In this case, I want students to see that this expression is basically two factors. The last step will be 3 times some number, the second factor being the value of the parentheses.

So, I write on the board **3 (?).** That is the big picture. Then we work our way in.

What is the big picture inside the parentheses?

$$\mathbf{\frac{12+4}{8} + \frac{2(35)}{12+2}}$$

This is addition. The different parts will simplify to two addends, and the last step will be finding the sum.

That leads us to zoom in on each addend.

$$\mathbf{\frac{12+4}{8}}$$

We will need to add 12 + 4 in order to find the dividend, then we can divide by 8. So this will simplify to $\frac{\mathbf{16}}{\mathbf{8}} = \mathbf{2}$

$$\frac{\mathbf{2(35)}}{\mathbf{12+2}}$$

This will also be a quotient, but we will need to simplify the dividend and the divisor before we can divide.

$$\frac{\mathbf{70}}{\mathbf{14}} = \mathbf{5}$$

Although we are actually simplifying (doing the calculations) from the inside out, we identified where to start by looking outside (at the big picture structure, what comes last) and working our way in.

$$\mathbf{3}\left[\frac{\mathbf{12+4}}{\mathbf{8}} + \frac{\mathbf{2(35)}}{\mathbf{12+2}}\right]$$

$$\mathbf{3}\left(\frac{\mathbf{16}}{\mathbf{8}} + \frac{\mathbf{70}}{\mathbf{14}}\right)$$

$$\mathbf{3(2+5)}$$

$$\mathbf{3(7)}$$

$$\mathbf{21}$$

Imagine a student trying to work through the “beast” above using only a step-by-step PEMDAS.

$$\mathbf{3}\left[\frac{\mathbf{12+4}}{\mathbf{8}} + \frac{\mathbf{2(35)}}{\mathbf{12+2}}\right]$$

Hmm, parentheses first. But where do I start inside them? Do I have to start with the parentheses around the 35? What is being divided by 8? (No parentheses are given, although the addition in the dividend needs to come first.) I guess I need to multiply next. 3 × 12?

Teaching tips

Affirm previous knowledge and deepen it

I don't ever say that PEMDAS is wrong, especially because so many people know it and it is technically accurate. I affirm what students know and explain that PEMDAS is often misunderstood or confusing, and that we are going to learn how to interpret expressions and make sense of them.

Start with numerical expressions and accessible, whole numbers

Until students are comfortable and fluent with symbols, and structure, don't add a further tax on their working memory with difficult numbers. Also, students will need to understand structure with numerical expressions before they will be able to understand them with algebraic expressions (ones that include variables).

Working Outside-In: Think alouds

I model a lot of think alouds with my students as we look at a complex expression and look for its structure, asking the same questions (What is the big picture? What will come last?). When I am gesturing or highlighting or circling parts of the expression on the board that we can all see, it is easier to refer to "this" while circling or gesturing to part of an expression. "This" is going to be a factor; at the end we will multiply it by 3.

Expand and Simplify

This routine was discussed in Part 1, but it is excellent for working on structure, because students are going in both directions, expanding the expression and simplifying it. They can see what the big picture is because that is where they started!

Number of the Day

The Number of the Day was also discussed in Part 1. As students get more fluent, push them to build more complex expressions. Use student expressions as examples for a Think Aloud.

PART 3: VARIABLES AND ALGEBRAIC EXPRESSIONS

When most students think of algebra, they think of math with letters, and this is often where a great deal of anxiety and confusion comes up. Understanding how and why we use letters in math is important to demystify algebra and to help students use variables correctly and strategically to communicate their thinking. In part 3 of this blog series, we will look at what variables are, some conventions around variables (and some complicating factors), and ways to help students gain confidence with algebraic expressions.

In Parts 1 and 2, we mainly focused on helping students gain confidence with the notation and structure of numerical expressions—ones that use constant numbers only (no variables). However, algebra is about finding and describing stable relationships between different quantities, and quantities can change.

A conceptually-focused approach to teaching algebra will start with relationships in a familiar context. For example:

My brother is 2 years younger than me.

This is a context and mathematical relationship everyone understands: the relationship is stable (my brother will always be two years younger, no matter what), but the amounts that the relationship is between (my age and my brother's age) will change.

Transforming this contextualized relationship into an algebraic function requires several things.

First, we need to identify what the quantities are that have a relationship. This can be tricky. The quantities themselves are not actually identified in the sentence above, which is an example of how we describe this relationship in everyday language ("people talk", to use the phrase coined by Bob Moses). What we are actually comparing is not me and my brother (we are not numbers and no mathematical function can transform me into my brother). However, the sentence is actually comparing two amounts. To translate it into "number talk" (another Bob Moses term), we could say

The age of my brother is 2 years less than my age.

It is ages that are being compared, not people!

Algebraic in-out tables can help to illustrate a relationship.

My Age	My Brother's Age
3 – 2 =	1
5 – 2 =	3
20 – 2 =	18
78 – 2 =	76

Because the relationship (in this case, a function, seen in red) stays the same, we can predict my brother's age based on any amount for my age—even ages I have not yet reached.

Looking at the table above, the relationship (-2) stays the same, but the ages can change. We use letters as variables to represent amounts that can change. In this case, the numbers in the left column (my age) can change, and the numbers in the right column (my brother's age) can change. So, we can choose a variable for each one. Let's use *m* for my age and *b* for my brother's age.

The specific equations in the table then become a more general statement of how the two amounts are related

$$m - 2 = b$$

This type of example is a good introduction to the use of variables as standing for real life amounts that can change and that can be in relationship to other amounts. In this case, variables represent something semi-concrete (an age of a person), but something that changes in a way we are familiar with, so that we don't want to tie it down to a single value.

Variable Shenanigans

Some notational and conceptual issues can come up that make variables confusing for students, even if they understand the basic concept.

First of all, while any letter (or symbol, for that matter) can be used as a variable, some are better choices than others. I have my students brainstorm some letters that would not make good variables, and to explain why. (Some examples include *o*, lowercase *l*, lowercase *t*, or other letters that can be mistaken for numbers or mathematical operators.) Sometimes these are used with a scripted font, for example, ℓ when it represents length in a geometric formula.

It makes sense for students to choose variables that will help them remember what the variable represents, with a few caveats. In the example above, I could not use the variable *a* for both ages, because my equation would have read

$$a - 2 = a$$

which is not true and doesn't indicate that the two ages are different. Sometimes students want to use two letters

$$AM - 2 = AB \text{ (as in, age of me and age of brother)}$$

If both letters are written in the same size, this is incorrect, as it indicates two variables multiplied together (*A* times *M*). However, it is common to differentiate with subscripts, as in this example

$$A_m - 2 = A_b$$

Since the subscripts are understood to provide an additional label for the variable letter, they are not seen as amounts themselves.

Some specific variables have conventional uses. For example, it is common to use *n* to represent an ordinal number (as in, the n^{th} in line). This is used in math for patterns and sequences and can be useful to teach explicitly.

Not all letters are variables!

Finally, variables as letters can be confusing because not all letters in math are variables.

For example, $x - 5 = 10$, x is not a variable. There is only one value that makes the equation true, so it can't vary. In this case, x is an "unknown". Solving for an "unknown" is often what students associate with algebra, which can limit their understanding of how a letter can function as a variable, as in, a generalized value that can vary.

Letters can also represent constant numbers. For example, *π* is not a variable. It is a Greek letter representing a constant irrational number that can't be fully written out. For most HSE students, *π* may be the only "constant represented by a letter" that they encounter, but this is common in science and higher level math: *g* for the acceleration due to gravity, *c* as the speed of light, *i* as an imaginary unit in complex numbers, etc.

More relevant to HSE level math, letters are used to represent functions. $f(x)$ is a tremendously confusing notation for math students, since f (often typeset in script) is naming a function, not a variable number. (This is one of the reasons I spend a lot of time teaching students to multiply variables without parentheses, so that we can make room for this notation to mean something different.)

Teaching tips

Start with actual variables and relationships, worry about unknowns later

Start the use of variables by exploring common mathematical relationships that students already understand. For example,

time and paycheck when working for $17 for hour

miles and hours when driving 60 mph on the highway

Note: Both the *CALM curriculum* and *EMPower materials* that it draws from use this approach in their units on linear algebra. Students will be writing linear mathematical functions that use two variables. They will learn that functions show a relationship between two amounts that can change.

Discuss aspects of variable choice and conventions as they arise

Later, students can learn how to solve for an unknown, which will allow them to use their functions to make predictions (i.e., How old will my brother be when I am 98? How old will I be when my brother is 87?) By substituting in a value for one of the variables in a relationship, they will have to use the function to solve for the other variable (now an unknown). Starting with one- and two-step functions will allow this to become intuitive and deepen students understanding of inverse operations before getting into any intense symbolic manipulation.

Extend understanding of properties of operations to variable expressions

To become truly fluent with algebraic expressions, students will need to extend their understanding of properties of operations to expressions with variables. As a prerequisite, students will need to understand these properties with constant numbers.

This understanding is built through a progression that goes from

concrete → representational → symbolic

For example, the distributive property is incredibly important for working with complex algebra expressions. However, if students only understand the distributive property in its most symbolic form

$$a(b + c) = ab + ac$$

it may seem like one of those meaningless rules that makes "algebra" so hard. First, students need to have a more intuitive understanding of what the distributive property actually is. At a basic level:

7 hands is the same as 2 hands and 5 hands.

If we think of "hands" as groups of five fingers, this gives us a way to multiply 7 hands times 5 fingers each in parts: 2 hands with 5 fingers each, plus 5 hands with 5 fingers each.

In other words, $7 \times 5 = 2 \times 5 + 5 \times 5$ (seven fives is the same as two fives and 5 fives)

Breaking up arrays using the area model of multiplication is another way to demonstrate the Distributive Property concretely, and then representationally as students build conceptual understanding. Read more on Area Models here.

Eventually, this is generalized. I can break up any factor when I am counting equal groups.

$$9 \times 6 = 6 \times 6 + 3 \times 6 = 6(6 + 3)$$

When students understand how and why this works with numerical expressions, then they are ready to extend it to algebraic expressions

$$a(3 + 7) = 10a = 3a + 7a$$

and so on.

Like everything else discussed in this series, gaining fluency with properties of operations is something that happens slowly over time. Don't teach all of the properties at once or expect students to learn them quickly; it is much more effective to weave this material and practice into your curriculum and to revisit it frequently with routines, homework, and quizzes.

A final note for teachers:

Teaching your students algebraic notation, expressions, and properties is an important part of helping students learn "the language of algebra," but it is just as important that they learn how that language is used to solve interesting and meaningful problems. A rigorous algebra curriculum will help students build this symbolic fluency while giving them plenty of opportunity to engage in collaborative algebraic reasoning about mathematical relationships in real life contexts. If you are looking for an intermediate to advanced adult math curriculum that builds in all aspects of rigor, check out information about CALM (Curriculum for Adults Learning Math) at https://terc.edu/calm.