The Answer Is Still the Same ...

It Doesn't Matter How You Got It!

A Comparison of US and Other Computation Methods for Math Teachers and Students from Various Backgrounds

Mary Jane Schmitt

The Answer is Still the Same... It Doesn't Matter How You Got It! was written by Mary Jane Schmitt, founder of the Adult Numeracy Center at TERC, and co-founder of the Adult Numeracy Network. The original version of this document was published in 1991.

Updates were made in 2006 and 2020.

Adult Numeracy Center

TERC
Because math and science build futures

Questions, comments, or requests for use of this publication may be directed to **adultnumeracy@terc.edu**.

Contents

Foreword

This booklet was first written by Mary Jane Schmitt in 1991 as the result of a teacher research project conducted with adult learners of math at the Community Learning Center in Cambridge, Massachusetts.

Since then, there has been a consistent request for copies of it. In that time also, the number of adult learners in the US from different backgrounds and educational traditions has grown enormously.

Since it fills a gap in the literature available to teachers of math to adults, we have worked with Mary Jane to produce an updated edition.

So read, enjoy, and discuss it with your students.

Andrew Pates
Peppercorn Books & Press
April 2006

Introduction to the First Edition

Adult basic education teachers learn a lot from their students. This booklet is a record of some of the things I learned from immigrant adults who came to math classes at the Community Learning Center in Cambridge, Massachusetts. It's important that anyone
teaching math to adults from other countries appreciate the fact that there are significant differences in the methods that people learn to compute around the world. Adult basic education teachers like to build on their students' strengths and backgrounds. Hopefully, this booklet will help you do that in math.

The adults in my classes came from well over twenty different countries, but most were from the Caribbean and Central America. What I discovered was that there seemed to be two groups as far as math methods go.

Some of the people had learned to do math sort of the same way we do it in the United States. These folks were from countries that had been colonized or whose formal educational institutions had been influenced by the United States or Great Britain (e.g., Jamaica or Puerto Rico.)

The other group came from countries, which had been influenced, by mainland Europe: France, Spain, Portugal, etc.

Hegemony rears its head in long division, too!

This booklet compares the "United States methods" with the 'European methods': According to my classroom-based research, I've classified countries according to the common math method. There are variations though, so make sure to ask each student how he or she learned math as a child. Often it depends on what country the teachers were from (i.e., an Irish monk in Haiti taught the British way rather than the European).

Mary Jane Schmitt
April 1991

Notes for a Revised Edition

Originally, I put this booklet together because I felt strongly that immigrant adults who attended adult education classes should be given credit for what they know, and not waste time replacing their tried and true arithmetic methods with others. I felt it was important for their teachers not only to be aware that notation and arithmetic algorithms differed by country or culture, but to also understand those various procedures and how they work. I still believe that, and for that reason, this original booklet still has value to US educated teachers who work with students from other countries.

However, the booklet could use some timely input for an expanded and updated version. In the next version, we would like to:

1. **Document math methods from more countries.** The information presented here is from interviews of a small sample of students from 16 countries. We would like to interview more students from additional countries and find out about other math areas as well.

2. **Sample the current population to see if the methods they are learning now have changed in anyway.** In other words, do students from the countries we surveyed in 1991 still learn to compute with these methods? Have the basic math curricula changed?

3. **Reflect more broadly the issues that face immigrant students in basic math or numeracy classes.** To be sure, it is not only paper and pencil computational methods that come into play. What are the cross language issues?

4. **Support the development of number and operation sense.** Fifteen years ago, the instructional emphasis in adult basic education math classes was on paper and pencil computational procedures. Students spent the majority of class time practicing computational algorithms. Much has changed since then. With the advent of the calculator and computers, the emphasis has shifted to building an adult's facility with estimation (to check for reasonableness of calculations) and with problem solving. The relative percentage of time spent on teaching long division or computation with unwieldy fractions has lessened. Calculators reduce the need to know how to divide $42,630 by 5 with paper and pencil methods. Most math educators would agree that the more essential skills are the number and operation sense to know that $40,000 split 5 ways is $8,000 -so one would expect an answer just a bit greater than $8,000. Knowledge of how to key in the numbers into a calculator (order does matter in division) is important as well. What are effective ways to support these skills?

We encourage you to interview students about their methods and share what you find.

Mary Jane Schmitt
April 2006

Math Methods by Country

US METHOD

United States

Puerto Rico

Jamaica

Ethiopia

Philippines

"EUROPEAN" METHOD

Haiti

Portugal (Azores)

Cape Verde

Cambodia

Vietnam

France

South American countries

- Brazil
- Colombia

Central American countries

- El Salvador
- Guatemala

Greece

Numbers

Numerals	English	French	Spanish	Khmer	Vietnamese
1	one	un, une	uno, una	muy	một
2	two	deux	dos	pi	hai
3	three	trois	tres	bey	ba
4	four	quarte	cuatro	bourn	bốn
5	five	cinq	cinco	pram	năm
6	six	six	seis	pram muy	sáu
7	seven	sept	siete	pram pi	bảy
8	eight	huit	ocho	pram bey	tám
9	nine	neuf	nueve	pram bourn	chín
10	ten	dix	diex	dap	mươi
100	one hundred	cent, centaine	cien(to)		
1,000	one thousand	mille	mil		
1,000,000	one million	un million	million		
1,000,000,000	one billion	un milliard	billon, mil millones		
1/2	one-half	la moitie	la mitad		
1/10	one-tenth	un dixieme	decimo		
1/100	one-hundreth	un centieme	centesimo		
1/1,000	one-thousandth	un millieme	millonesimo		

In the US, a **billion** means a **thousand million**.
In the UK, a **billion** means a **million, million**.

Notation

Whole Numbers

In the US:	1,000 means one thousand
In Europe:	1.000 means one thousand

In the United States we use commas to mark off a whole number in groups of three digits for thousands, millions, billions, etc. Two million, five hundred thousand is written 2,500,000.

In Europe this would be written as 2.500.000. The point(.) is used to separate the number into groups of three digits.

Decimals

In the US:	1,000 means one thousand
In Europe:	1.000 means one thousand

In the United States we use a point to separate a whole number from a decimal fraction. In US textbooks, **six tenths** is written either as **.6** or **0.6**.

In Europe a comma is used instead of our decimal point. The zero is always written in the whole number place **(0,6)**.

Money

Decimals are used in many countries to separate the main monetary unit from the hundredth part of that unit.

Dollars and cents in the US:	$3.25
Gourdes and centimes in Haiti:	3,25 G
Francs and centimes in France:	3,25 F (Now Euros and Euro cents)

Whole Number Computation

Addition and Multiplication

Addition and Multiplication computational methods are essentially the same. However, in the United States, we are taught to put the number we carry above the next column of numbers; in other countries, the number carried may be put **over to the right** or not written at all.

US	Europe
$\overset{1\ 2}{\ }$ 256 $\overset{1}{\ }$8,899 + 2,015 **11,170**	256 8.899 211 + 2.015 **11.170**

Multiplication is sometimes written horizontally.

US	Europe
$\overset{1}{\ }$52 x 36 312 156 **1,872**	 52 x 36 312 156 = **1.872**

Subtraction

Subtraction, European style, varies significantly from the Unites States method. In the US the concept of **borrowing** is used when subtracting a larger amount from a smaller amount.

Borrowing or renaming is **not** used in French and Spanish speaking countries. Rather, the concept of adding to each amount is taught.

US method of subtracting 53 – 17

$\overset{4\ 1}{\ }$53
– 17

Thought process...

1. We can't take 7 from 3 so we borrow 1 (ten) from 5 (tens), leaving 4 (tens). Put the 1 in front of the 3 calling it 13.

$\overset{4\ 1}{\ }$53
– 17
36

2. 7 from 13 leaves 6.
   1 from 4 leaves 3.

European method of subtracting 53 – 17

	Thought process...
$\overset{1}{53}$ $-\ 17$	1. We can't take 7 from 3 so we make the 3 into a 13.
$\overset{1}{53}$ $-\ 17$ **6**	2. 7 from 13 leaves 6.
Becomes 2 $\overset{1}{53}$ $-\ 17$ **36**	3. Now, add 1 to the bottom of the next column (1 + 1 = 2). 4. 2 from 5 leaves 3.

The student using this method should be reminded that she/he should **only add 1** to the bottom number when she/ he puts a one in front of the number on top.

Important implications:

Students who use this European method often have trouble understanding an explanation of borrowing in subtraction of fractions, decimals and measurements because the concept is totally foreign.

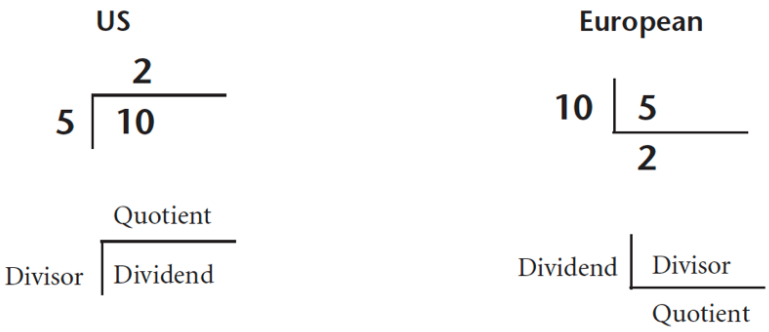
More subtraction problems, European style ...

$\overset{111}{4000}$ $+\ \overset{269}{1583}$ **2417**	$\overset{11}{4000}$ $+\ \overset{69}{1583}$ **3117**

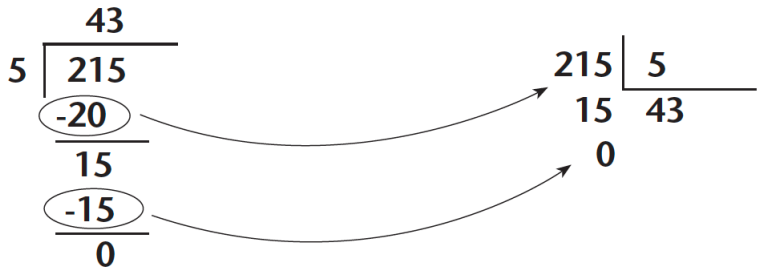
Division

The European method of division differs from the US method in four ways.

1. The divisor and dividend are written in reverse positions, and the quotient is written beneath the divisor in the European method, but above the dividend in the US method.

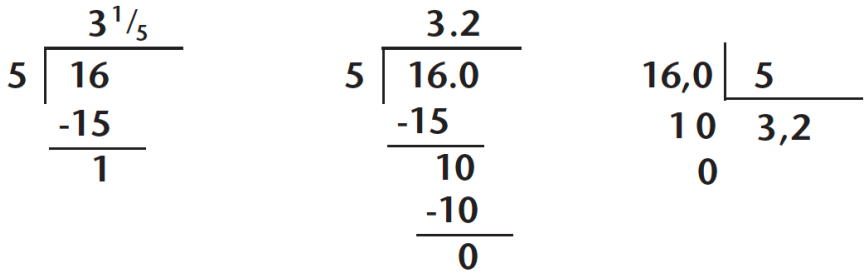


2. The multiplication step is not written down.



3. With two and three digit divisors, the multiplication and subtraction steps are done as one step. (I still can't do this after lots of practice.)

4. When there is a remainder, the European method never expresses it as a fraction; a comma with zeros is added to express the remainder as a decimal.



In European division $5 \overline{)\,42630}$ becomes $42630 \,\big|\, 5$

Student does on paper	Thought process	Comments
42630 | 5 8	42/5 = 8	Answer is written under the divisor.
42630 | 5 2 8	8 × 5 = 10 42 - 40 = 2	The **multiplication** and **subtraction** steps are **never written down.** Only the remainder is written.
42630 | 5 26 8	Bring down the next number 6	
42630 | 5 26 85	26/5 = 5 5 × 5 = 25 26 - 25 = 1	
42630 | 5 26 85 1		
^{1}42630 | 5 26 85 13	Bring down the next number 3	

Student does on paper	Thought process
42630 | 5 26 852 13	13/5 = 2 2 × 5 = 10
42630 | 5 26 852 13 3	13 - 10 = 3
42630 | 5 26 852 13 30	Bring down the next number 0
42630 | 5 26 8526 13 30	30/15 = 6 5 × 6 = 30
42630 | 5 26 8526 13 30 0	30 - 30 = 0

Try to see how far you can get using the European method. Problems are completed on the back of this page. Remember to use a comma as a decimal marker instead of a point.

Problems are completed on the next page.

12 | 4	11 | 5	128 | 4
1374 | 4	238 | 5	420 | 5
836 | 15		278 | 12
7365 | 28		846 | 40

```
12 | 4
   | 3
```

```
11 | 5
1,0| 2,2
 0 |
```

```
128 | 4
 08 | 32
  0 |
```

```
1374 | 4
  17 | 345,5
  14 |
 2,0 |
   0 |
```

```
238 | 5
 38 | 47,6
3,0 |
```

```
420 | 5
 20 | 84
  0 |
```

```
836 | 15
 86 | 55,733
11,0|
 50 |
  50 ← may keep adding zeros
```

```
278 | 12
 36 | 23
```

```
7365 | 28
 176 | 263,03
  85 |
1,00 |
  16 |
```

```
846 | 40
 46 | 21,15
6,0 |
200 |
```

Decimals

Addition, subtraction, multiplication

Addition, **subtraction** and **multiplication** are done similarly in both methods, except keep in mind that **the decimal point becomes a comma,**

US		Europe
3.6 2.45 + 18. 24.05	**RULE:** To add or subtract line up the decimal markers	3,6 2,45 + 18, 24,05
3.6 – 2.43 1.17		3,6 – 2,43 1,17
3.6 x .25 180 72 .900 = .9	**RULE:** To multiply, total the number of decimal places.	3,6 x 0,25 180 72 ,900 = ,9

Division

Division rules are quite different.

Case#1 A decimal divided by a whole number

US			Europe
7.48 2 ⟌14.96 14 09 8 16 16 0	⟸ **RULE** Place the point directly above the point in the dividend, then divide.	**RULE** ⟹ Start dividing, put the comma in the answer when you come to it in the dividend.	14,96 ⟌ 2 0 9 7,48 16 0

Division of decimals

Case #2 A decimal divided by a decimal.

US

```
      66.2
.02 | 1.324
      12
      12
      12
      04
       4
       0
```

Europe

Three decimal places ⇨

```
1,324 | 0,020
  12    66,2
  04,0
     0
```

⇦ Only two decimal places, so add on 1 zero to make 3 decimal places

RULE
Move the point in the divisor to the end, and move the point in the dividend the same number of places.

Place the point directly above the quotient, then divide.

RULE
Equalize the number of decimal places, in this case, by adding a zero in the divisor.

Now drop the comma completely. Do the division.

If you need to continue the division, add a comma and a zero.

Another European example

```
0,2 | 0,025
```

1. Add two zeros to "even it up".

```
0,200 | 0,025
```

2. Now, drop the commas.

```
0200 | 0025
```

3. Do the division.

```
0200 | 0025
       8
```

Fractions

Compare the two methods for fraction **addition**.

US

$$\begin{array}{r} \frac{1}{8} \\ +\frac{3}{8} \\ \hline \frac{4}{8} = \frac{1}{2} \end{array}$$

All fraction operations are written **horizontally**. The fraction is usually left unsimplified.

⟹

Europe

$$\frac{1}{8} + \frac{3}{8} = \frac{1+3}{8} = \frac{4}{8}$$

$$\begin{array}{rrcl} & \frac{1}{8} & \begin{array}{c}\times 3\\ \times 3\end{array} & = \frac{3}{24} \\ + & \frac{5}{6} & \begin{array}{c}\times 4\\ \times 4\end{array} & = \frac{20}{24} \\ \hline & & & \frac{23}{24} \end{array}$$

⟸ **RULE**

1. Find the **lowest common denominator.**
2. Rename each fraction in highest terms.
3. **Add** the **numerators, keep** the same **denominator.**

RULE ⟹

1. Find the **lowest common denominator.**
2. **Divide** the LCD by the **first denominator (24 : 8 = 3)** and **multiply** the result by the **first numerator (3 x 1 = 3)**. **Divide** the LCD by the **second denominator (24 = 6 = 4)** and **multiply** the result by the **second numerator.**
3. Evaluate the numerator.

$$\frac{1}{8} + \frac{5}{6} = \frac{}{24}$$

$$\frac{1}{8} + \frac{5}{6} = \frac{(3 \times 1) + (4 \times 5)}{24}$$

$$= \frac{3 + 20}{24}$$

$$= \frac{23}{24}$$

Finding the lowest common denominator European style

This method uses the idea of extracting common factors.

$\frac{1}{8} + \frac{5}{6}$

Write the denominators on top. ⇨

Factors are extracted and listed here. ⇨

	8	6
2	4	3
2	2	3
2	1	3
3	-	1
24		

⇦ Remaining factors are kept track of here.

- Multiply the list of factors. This product is the LCD. (2 x 2 x 2 x 3 = 24)

Another example

$\frac{1}{15} + \frac{7}{10} + \frac{3}{16}$

	15	10	16
5	3	2	16
3	1	2	16
2	-	1	8
8	-	-	1
240			

(5 x 3 x 2 x 8 = 240)
The LCD is 240.

Fractions: Some general differences to be aware of

1. In the European method, mixed numbers are generally changed to improper fractions for all four operations.

Addition: $4\frac{1}{2} + 2\frac{3}{5} = \frac{9}{2} + \frac{13}{5} = (5 \times 9) + (13 \times 2) = \frac{45+26}{10} = \frac{71}{10}$

Subtraction: $4\frac{1}{2} - 2\frac{3}{5} = \frac{9}{2} - \frac{13}{5} = (5 \times 9) - (13 \times 2) = \frac{45-26}{10} = \frac{19}{10}$

Multiplication: $4\frac{1}{2} \times 2\frac{3}{5} = \frac{9}{2} \times \frac{13}{5} = \frac{117}{10}$

Division: $4\frac{1}{2} \times 2\frac{3}{5} = \frac{9}{2} \times \frac{5}{13} = \frac{45}{26}$

2. In the European method, all fractions operations are done horizontally. In the US method, adding and subtracting are set up vertically, whereas multiplication and division are done horizontally.

3. Canceling, in fraction multiplication, is usually not done in the **European** method.

US

$$\frac{\cancel{4}^{1}}{\cancel{2}5^{5}} \times \frac{\cancel{1}5^{3}}{\cancel{1}6_{4}} = \frac{3}{20}$$

Europe

$$\frac{4}{25} \times \frac{15}{16} = \frac{60}{400}$$

4. Students who went to school in countries using the European are generally more confident computing with decimals than with fractions because countries using the metric system use decimals in measurement and rarely use fractions for everyday math.

Measurement

Systems of measurement vary significantly in every country. Usually a country has an old fashioned system and a modern system (metric). This applies to length, weight, capacity, volume, area, and money.

	US English	Metric
Length	inch, foot, yard, mile	meter, centimeter, millimeter, kilometer
Weight	ounce, pound, ton	gram, kilogram, milligram
Liquid Capacity	fluid ounce, cup, pint, quart, gallon	liter, milliliter
Area	square inch, foot, yard	m^2, cm^2, mm^2, km^2
Volume	cubic inch, cubic foot, cubic yard	cc^3, cm^3, m^3
Time	same in all countries	

When you teach the English system of measurement, make sure you have your students use rulers, yardsticks, measuring tapes, scales and containers to really get the idea of what's going on.

Country	Monetary Unit
United States, Puerto Rico	US dollar, cents
France, Greece, Portugal	Euro, euro cents
Haiti	Gourde, centime
Cambodia	Riel
Vietnam	Dong
Brazil	Real (formerly cruzeiro), centavos
Colombia	Peso, centavos
El Salvador	US dollar (formerly colón)
Guatemala	Quetzal, centavos
Jamaica	Jamaican dollar
Mexico	Peso, centavo
Ethiopia	Birr, santim
Philippines	Peso, centavos
Cape Verde	Escudo, centavos

Glossary

English	French	Haitian Kreyol	Spanish	Vietnamese
addition	l'addition	egal	la adición	phép công
plus (+)	plus	plis	más	
answer, result	réponse	repons	respuest, el resultado	
difference	différence, reste, excés	diferans	la diferencia	
carrying	retenue			
equals (=)	égale	egal	igal	bằng
subtraction	la soustraction	soustraksyon	substracción, la resta	phép trừ
minus (–)	moins	mwans	menos	
multiplication	la multiplication	miltiplikasyon	la multiplicación	phép nhân
product	produit	prodit	el producto	
times, by (×)	par, multiplié par	miltipliye pa	por, multiplicado	
division	la division	divisyon	la división	phép chia
quotient	le quotient	kosyan	el cociente	
dividend	dividende	dividann	dividendo	
divisor	diviseur	divize	divisor	
remainder	reste	res	el residuo	
divided by (/ or ÷)	entre	divize pa	divida por	
average	la moyenne	mwayén	el promedio	trung bình
whole number	nombre entier	nonm antye	número entero	số nguyên
decimal	nombre decimal	desimal	el decimal	thập phân
fraction	fraction	fraksyon	la fracción	
square (shape & exponent)	carré	kare	el cuadrado	
square root	racine carré	rasin kare	raíz cuadrada	
ration & proportion	rapport et proportions (regle detrois)	rapo pwoposyòn	la razón y proporción	
percent	pour cent	pousantaj	porciento	phần trăm
lowest common denominator (LCD)	plus petit multiple commune (PGCD)	pi piti faktè komen	mínimo denominador común	
odd	impair	enpè	impar	
even	pair	pè	par	

Student Interview

Some ideas about interviewing a student to find out how s/he does math in his/her country.

Name:
Country:
Notation:

Languages:
School language:

How do you write?

One thousand _____

One million _____

One billion _____

One tenth (1/10) _____

One hundredth (1/100) _____

How do you do the following problems?

Addition: 582 + 138

Subtraction: 582 – 138

Multiplication: 346 × 24

Division: $6\overline{)205}$ or $205\overline{)6}$

Fractions: How do you write 3 and five tenths?

Decimals: How do you write 3 and five tenths?

Percents: How do you find 20% of 250?
30 is what percentage of 80?

What are the units of measurement in your country?

Length (e.g., cloth):
Liquid (e.g., milk, gasoline)
Area: (e.g., farmland)
Money:

Tell me how you learned multiplication tables in your country.